ISS0031 Modeling and Identification

Juri Belikov

Department of Computer Control,
Tallinn University of Technology

jbelikov@cc.ic.ee

September 5, 2014

Tallinn University of Technology

Department of Computer Control

Institute of Cybernetics

- Associate Professor
- **Room**: U02-320
- From 9AM until 12AM
- **e-mail**: juri.belikov@ttu.ee

- Researcher
- **Room**: CYB-325
- From 13PM until ...
- **e-mail**: **jbelikov@cc.ioc.ee**

Alpha Control Laboratory

- Department of Computer Control, Tallinn University of Technology
- U02-301a
- Established in the middle of 2013
- Education and Research
- Research focus: computational/artificial intelligence based methods, fractional calculus
- `http://a-lab.ee`

Course code	ISS0031
Subject title	Modeling and Identification
Subject title (in estonian)	Modelleerimine ja Identifitseerimine
Lecturer	Juri Belikov
Course volume ECTS	5
Stationary study (weekly hours)	lectures: 2, exercises: 2
Assessment form	*examination*
Teaching semester	autumn
Official working language	English

Where to find: http://**a-lab.ee**/edu/node/457

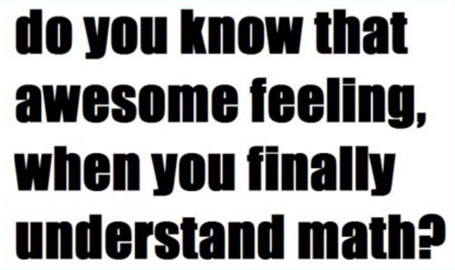
What to find: material, schedule, etc.

Recommended preparation (expected knowledge):

- Linear Algebra (YMA3710)
- basics of Mathematical Analysis (YMM3731)
- knowledge of programming languages (e.g., MATLAB or *Mathematica*) is useful
- basic knowledge of controls concepts (at the level of ISS0010 and ISS0021) is helpful.

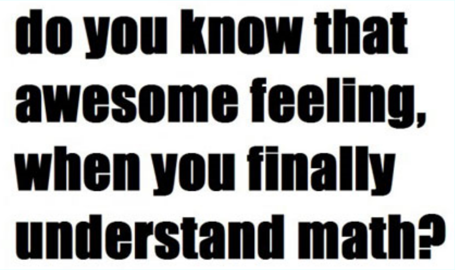
Recommended preparation (expected knowledge):

- Linear Algebra (YMA3710)
- basics of Mathematical Analysis (YMM3731)
- knowledge of programming languages (e.g., MATLAB or *Mathematica*) is useful
- basic knowledge of controls concepts (at the level of ISS0010 and ISS0021) is helpful.



Recommended preparation (expected knowledge):

- Linear Algebra (YMA3710)
- basics of Mathematical Analysis (YMM3731)
- knowledge of programming languages (e.g., MATLAB or *Mathematica*) is useful
- basic knowledge of controls concepts (at the level of ISS0010 and ISS0021) is helpful.



The **final** grade consists of two parts:

- Test – 40% (2 assignments)
- Final Project – 60% (7-10min. project proposal is due on **October 17th** in class presentation).

A project has to be **self-sufficient**, i.e., it has to contain:

- ✓ brief introduction,
- ✓ description of a problem,
- ✓ solution of a problem,
- ✓ examples/practical results,
- ✓ list of references.

The following two types of projects are possible:

1. Solution of a **research problem** relevant to the student's area of interest.
2. **Independent study** of a topic not covered in the course (e.g., reading a scientific article or book chapter).

- Application of linear programming in game theory
- Survey on algebraic framework of differential forms
- A realization problem (input-output to state-space)
- Implementation of scientific results in *Mathematica* or MATLAB environments
- Time scales theory based toolbox for MATLAB
- Survey on structural properties of linear switched
- Survey on networked control systems
- Modeling a laboratory object
- Modeling and implementation of fractance networks for control applications

Overview of the Course

Schedule

Class	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Topic																

Test & Co

Mathematical Programming

Control Theory

Fractional-order Calculus

Neural Networks

Practice

Any questions about *organizing* part of the course?

"*Per aspera ad astra.*" Lucius Annaeus Seneca

"*Through hardships to the stars.*"

A phone dealer goes to the wholesale market with 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of type P_1 and type P_2 are suitable. The cost price of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be sold for 325 EUR, while phone of the type P_2 can be sold for 265 EUR. Within the available amount of money he would like to make maximum profit. His problem is to find out how many type P_1 and type P_2 phones should be purchased so to get the maximum profit.

A phone dealer goes to the wholesale market **with** 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of type P_1 and type P_2 are suitable. The cost price of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be sold for 325 EUR, while phone of the type P_2 can be sold for 265 EUR. Within the available amount of money he would like to make maximum profit. His problem is to find out how many type P_1 and type P_2 phones should be purchased so to get the maximum profit.

A phone dealer goes to the wholesale market **with** 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of **type** P_1 and type P_2 are suitable. The cost price of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be sold for 325 EUR, while phone of the type P_2 can be sold for 265 EUR. Within the available amount of money he would like to make maximum profit. His problem is to find out how many type P_1 and type P_2 phones should be purchased so to get the maximum profit.

A phone dealer goes to the wholesale market **with** 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of **type** P_1 and type P_2 are suitable. The **cost price** of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be sold for 325 EUR, while phone of the type P_2 can be sold for 265 EUR. Within the available amount of money he would like to make maximum profit. His problem is to find out how many type P_1 and type P_2 phones should be purchased so to get the maximum profit.

A phone dealer goes to the wholesale market **with** 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of **type** P_1 and type P_2 are suitable. The **cost price** of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be **sold** for 325 EUR, while phone of the type P_2 can be **sold** for 265 EUR. Within the available amount of money he would like to make maximum profit. His problem is to find out how many type P_1 and type P_2 phones should be purchased so to get the maximum profit.

A phone dealer goes to the wholesale market **with** 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of **type** P_1 and type P_2 are suitable. The **cost price** of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be **sold** for 325 EUR, while phone of the type P_2 can be **sold** for 265 EUR. Within the available amount of money he would like to make **maximum profit**. His problem is to find out how many type P_1 and type P_2 phones should be purchased so to get the maximum profit.

A phone dealer goes to the wholesale market **with** 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of **type** P_1 and type P_2 are suitable. The **cost price** of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be **sold** for 325 EUR, while phone of the type P_2 can be **sold** for 265 EUR. Within the available amount of money he would like to make **maximum profit**. His problem is to find out **how many** type P_1 and type P_2 phones should be purchased so to get the maximum profit.

A phone dealer goes to the wholesale market **with** 1500 EUR to purchase phones for selling. In the market there are various types of phones available. From quality point of view, he finds that the phone of **type** P_1 and type P_2 are suitable. The **cost price** of type P_1 phone is 300 EUR/item and that of type P_2 is 250 EUR/item. He knows that one phone of the type P_1 can be **sold** for 325 EUR, while phone of the type P_2 can be **sold** for 265 EUR. Within the available amount of money he would like to make **maximum profit**. His problem is to find out **how many** type P_1 and type P_2 phones should be purchased so to get the maximum profit.

Table: Small summary

	P_1	P_2
Outcome	300	250
Income	325	265

P_1	P_2	**Investment**	**Amount after sale**	**Profit**
0	6	$0 \cdot 300 + 6 \cdot 250 = 1500$	$0 \cdot 325 + 6 \cdot 265 = 1590$	$1590 - 1500 = 90$

P_1	P_2	**Investment**	**Amount after sale**	**Profit**
0	6	$0 \cdot 300 + 6 \cdot 250 = 1500$	$0 \cdot 325 + 6 \cdot 265 = 1590$	$1590 - 1500 = 90$
1	4	$1 \cdot 300 + 4 \cdot 250 = 1300$	$1 \cdot 325 + 4 \cdot 265 = 1385$	85

P_1	P_2	**Investment**	**Amount after sale**	**Profit**
0	6	$0 \cdot 300 + 6 \cdot 250 = 1500$	$0 \cdot 325 + 6 \cdot 265 = 1590$	$1590 - 1500 = 90$
1	4	$1 \cdot 300 + 4 \cdot 250 = 1300$	$1 \cdot 325 + 4 \cdot 265 = 1385$	85
2	3	$2 \cdot 300 + 3 \cdot 250 = 1350$	$2 \cdot 325 + 3 \cdot 265 = 1445$	95

P_1	P_2	**Investment**	**Amount after sale**	**Profit**
0	6	$0 \cdot 300 + 6 \cdot 250 = 1500$	$0 \cdot 325 + 6 \cdot 265 = 1590$	$1590 - 1500 = 90$
1	4	$1 \cdot 300 + 4 \cdot 250 = 1300$	$1 \cdot 325 + 4 \cdot 265 = 1385$	85
2	3	$2 \cdot 300 + 3 \cdot 250 = 1350$	$2 \cdot 325 + 3 \cdot 265 = 1445$	95
3	2	$3 \cdot 300 + 2 \cdot 250 = 1400$	$3 \cdot 325 + 2 \cdot 265 = 1505$	105

P_1	P_2	**Investment**	**Amount after sale**	**Profit**
0	6	$0 \cdot 300 + 6 \cdot 250 = 1500$	$0 \cdot 325 + 6 \cdot 265 = 1590$	$1590 - 1500 = 90$
1	4	$1 \cdot 300 + 4 \cdot 250 = 1300$	$1 \cdot 325 + 4 \cdot 265 = 1385$	85
2	3	$2 \cdot 300 + 3 \cdot 250 = 1350$	$2 \cdot 325 + 3 \cdot 265 = 1445$	95
3	2	$3 \cdot 300 + 2 \cdot 250 = 1400$	$3 \cdot 325 + 2 \cdot 265 = 1505$	105
4	1	$4 \cdot 300 + 1 \cdot 250 = 1450$	$4 \cdot 325 + 1 \cdot 265 = 1565$	115

P_1	P_2	**Investment**	**Amount after sale**	**Profit**
0	6	$0 \cdot 300 + 6 \cdot 250 = 1500$	$0 \cdot 325 + 6 \cdot 265 = 1590$	$1590 - 1500 = 90$
1	4	$1 \cdot 300 + 4 \cdot 250 = 1300$	$1 \cdot 325 + 4 \cdot 265 = 1385$	85
2	3	$2 \cdot 300 + 3 \cdot 250 = 1350$	$2 \cdot 325 + 3 \cdot 265 = 1445$	95
3	2	$3 \cdot 300 + 2 \cdot 250 = 1400$	$3 \cdot 325 + 2 \cdot 265 = 1505$	105
4	1	$4 \cdot 300 + 1 \cdot 250 = 1450$	$4 \cdot 325 + 1 \cdot 265 = 1565$	115
5	0	$5 \cdot 300 + 0 \cdot 250 = 1500$	$5 \cdot 325 + 0 \cdot 265 = 1625$	125

P_1	P_2	**Investment**	**Amount after sale**	**Profit**
0	6	$0 \cdot 300 + 6 \cdot 250 = 1500$	$0 \cdot 325 + 6 \cdot 265 = 1590$	$1590 - 1500 = 90$
1	4	$1 \cdot 300 + 4 \cdot 250 = 1300$	$1 \cdot 325 + 4 \cdot 265 = 1385$	85
2	3	$2 \cdot 300 + 3 \cdot 250 = 1350$	$2 \cdot 325 + 3 \cdot 265 = 1445$	95
3	2	$3 \cdot 300 + 2 \cdot 250 = 1400$	$3 \cdot 325 + 2 \cdot 265 = 1505$	105
4	1	$4 \cdot 300 + 1 \cdot 250 = 1450$	$4 \cdot 325 + 1 \cdot 265 = 1565$	115
5	0	$5 \cdot 300 + 0 \cdot 250 = 1500$	$5 \cdot 325 + 0 \cdot 265 = 1625$	**125**

Decision: 5 phones of type P_1 should be purchased.

Mathematical programming problem can be written in the general form as

$$
\begin{aligned}
g_1(x_1, x_2, \dots, x_n) &\le 0 \\
g_2(x_1, x_2, \dots, x_n) &\le 0 \\
&\vdots \\
g_m(x_1, x_2, \dots, x_n) &\le 0 \\
(x_1, x_2, \dots, x_n) &\in S \subset \mathbb{R}^n
\end{aligned}
$$

Definition

The function to be maximized

$$z = f(x_1, x_2, \dots, x_n) \to \max$$

or minimized

$$z = f(x_1, x_2, \dots, x_n) \to \min$$

is called the **objective function**.

Definition

The limitations on resources which are to be allocated among various competing variables are in the form of equations or inequalities and are called **constraints** or **restrictions**.

Definition

A **linear programming problem** may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints.

The standard maximum problem can be stated as: Find a vector $x = (x_1, \dots, x_n)^T \in \mathbb{R}^n$, to maximize

$$z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

subject to the constraints

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &\leq b_1 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &\leq b_m \end{aligned}$$

and

$$x_1 \geq 0, x_2 \geq 0, \dots, x_n \geq 0.$$

Definition

A vector x for the optimization problem is said to be **feasible** if it satisfies all the constraints.

Definition

A vector x is **optimal** if it feasible and optimizes the objective function over feasible x.

Definition

A linear programming problem is said to be **feasible** if there exist a feasible vector x for it; otherwise, it is said to be **infeasible**.

Lemma

Every linear programming problem is either bounded feasible, unbounded feasible, or infeasible.

Definition

A vector x for the optimization problem is said to be **feasible** if it satisfies all the constraints.

Definition

A vector x is **optimal** if it feasible and optimizes the objective function over feasible x.

Definition

A linear programming problem is said to be **feasible** if there exist a feasible vector x for it; otherwise, it is said to be **infeasible**.

Lemma

Every linear programming problem is either bounded feasible, unbounded feasible, or infeasible.

Definition

A vector x for the optimization problem is said to be **feasible** if it satisfies all the constraints.

Definition

A vector x is **optimal** if it feasible and optimizes the objective function over feasible x.

Definition

A linear programming problem is said to be **feasible** if there exist a feasible vector x for it; otherwise, it is said to be **infeasible**.

Lemma

Every linear programming problem is either bounded feasible, unbounded feasible, or infeasible.

Definition

A vector x for the optimization problem is said to be **feasible** if it satisfies all the constraints.

Definition

A vector x is **optimal** if it feasible and optimizes the objective function over feasible x.

Definition

A linear programming problem is said to be **feasible** if there exist a feasible vector x for it; otherwise, it is said to be **infeasible**.

Lemma

Every linear programming problem is either bounded feasible, unbounded feasible, or infeasible.

Suppose that

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \end{bmatrix},$$

then linear programming problem can be rewritten in the **standard** or **canonical** matrix form as

Standard	**Canonical**
$z = CX \to \max(\min)$	$z = CX \to \max(\min)$
$AX \leq B$	$AX = B$
$X \geq 0$	$X \geq 0$

The **formulation** involves the following 3 steps:

1. identify the **decision variables** to be determined and express them in terms of algebraic symbols such as $x_1, x_2, \dots, x_n$;
2. identify the **objective** which is to be optimized (maximized or minimized) and express it as a linear **function** of the above defined decision variables;
3. identify all the **limitations** in the given problem and then express them as linear equations or inequalities in terms of above defined decision variables.

The **formulation** involves the following 3 steps:

1. identify the **decision variables** to be determined and express them in terms of algebraic symbols such as $x_1, x_2, \ldots, x_n$;

2. identify the **objective** which is to be optimized (maximized or minimized) and express it as a linear **function** of the above defined decision variables;

3. identify all the **limitations** in the given problem and then express them as linear equations or inequalities in terms of above defined decision variables.

The **formulation** involves the following 3 steps:

1. identify the **decision variables** to be determined and express them in terms of algebraic symbols such as $x_1, x_2, \ldots, x_n$;
2. identify the **objective** which is to be optimized (maximized or minimized) and express it as a linear **function** of the above defined decision variables;
3. identify all the **limitations** in the given problem and then express them as linear equations or inequalities in terms of above defined decision variables.

Solving linear programming problem is nothing but determining the values of decision variables that maximizes or minimizes the given effective measure satisfying all the constraints.

- Graphical method.
- Analytical method or trial and error method.
- Simplex method.
- Big-M method.
- Two phase simplex method.
- Dual simplex method.
- Revised simplex method.

A company manufactures two types of products P_1 and P_2 and sells them at a profit of 2 EUR and 3 EUR, respectively. Each product is processed on two machines M_1 and M_2. P_1 requires 1 minute of processing time on M_1 and 2 minutes on M_2, type P_2 requires 1 minute on M_1 and 1 minute on M_2. The machine M_1 is available for not more than 6 hours and 40 minutes, while machine M_2 is available for 10 hours during one working day.

Machine	Processing time		Available time
	P_1	P_2	
M_1	1	1	400
M_2	2	1	600
Profit	2	3	

Problem

Maximize the profit of the company.

A company manufactures **two types** of products P_1 and P_2 and **sells** them at a profit of 2 EUR and 3 EUR, respectively. Each product is processed on **two machines** M_1 and M_2. P_1 requires 1 minute of **processing time** on M_1 and 2 minutes on M_2, type P_2 requires 1 minute on M_1 and 1 minute on M_2. The machine M_1 is **available** for not more than 6 hours and 40 minutes, while machine M_2 is **available** for 10 hours during one working day.

Machine	Processing time		Available time
	P_1	P_2	
M_1	1	1	400
M_2	2	1	600
Profit	2	3	

Problem

Maximize the profit of the company.

- x_1 – the number of products of type P_1,
- x_2 – the number of products of type P_2.

The profit on selling:

- x_1 units of type P_1 is 2 EUR per product $\Longrightarrow 2x_1$,
- x_2 units of type P_2 is 3 EUR per product $\Longrightarrow 3x_2$.

Therefore, total profit on selling x_1 units of type P_1 and x_2 units of type P_2 is given by (objective function)

$$z = 2x_1 + 3x_2.$$

Machine M_1 takes 1 minute time on type P_1 and 1 minute time on type P_2
$\Longrightarrow$ the total number of minutes required on machine M_1 is given by $x_1 + x_2$.
Availability: not more than 6 hours and 40 minutes.

$$x_1 + x_2 \leq 400$$

The total number of minutes required on machine M_2 is given by $2x_1 + x_2$.
Availability: not more than 10 hours.

$$2x_1 + x_2 \leq 600$$

CANNOT produce negative quantities

$$x_1 \geq 0, x_2 \geq 0.$$

The problem is to find x_1 and x_2 which maximize the objective function z and can be formally written as

$$\begin{aligned} z = 2x_1 + 3x_2 &\to \max \\ x_1 + x_2 &\leq 400 \\ 2x_1 + x_2 &\leq 600 \\ x_1 \geq 0, x_2 &\geq 0. \end{aligned}$$

Let there be three different **types** of food F_1, F_2, F_3, that supply varying **quantities** of 2 nutrients N_1, N_2.

Suppose a person has decided to make an individual plan to improve the health.

We know that 400 g and 1 kg are the **minimum** daily requirements of nutrients N_1 and N_2, respectively. Moreover, the corresponding unit of food F_1, F_2, F_3 **costs** 2, 4 and 3 EUR, respectively. Finally, we know that

- one unit of food F_1 contains 20 g of nutrient N_1 and 40 g of nutrient N_2;
- one unit of food F_2 contains 25 g of nutrient N_1 and 62 g of nutrient N_2;
- one unit of food F_3 contains 30 g of nutrient N_1 and 75 g of nutrient N_2.

The given information can be arranged in the form of the following table:

Nutrients	Food			Requirement/day
	F_1	F_2	F_3	
N_1	20	25	30	400
N_2	40	62	75	1000
Price	2	4	3	

Problem

Supply the required nutrients at **minimum** cost.

Tables are good! Why? → see the next slide ...

Let x_i for $i = 1, 2, 3$ be the number of units of food F_i to be purchased per day. The problem can be formally written as

$$
\begin{gathered}
z = 2x_1 + 4x_2 + 3x_3 \to \min \\
20x_1 + 25x_2 + 30x_3 \geq 400 \\
40x_1 + 62x_2 + 75x_3 \geq 1000 \\
x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.
\end{gathered}
$$

- 3 **warehouses** W_i for $i = 1, \ldots, 3$ with commodity of the same type in **amount** of 200, 300, 450 units
- 4 **consumers** C_j for $j = 1, \ldots, 4$ who want to **receive** at least 150, 300, 150, 200 units of the commodity.

The **cost** of transporting one unit of the commodity from warehouse W_i to consumer C_j together with available information are summarized in the following table:

Warehouse	Consumers				Reserve
	C_1	C_2	C_3	C_4	
W_1	3	2	7	1	200
W_2	1	4	5	2	300
W_3	2	7	4	3	450
Requirement	150	300	150	200	

Problem

Meet the consumer requirements at **minimum** transportation cost.

The total transportation cost is

$$z = 3x_{11} + 2x_{12} + 7x_{13} + x_{14} + x_{21} + 4x_{22} + \\ + 5x_{23} + 2x_{24} + 2x_{31} + 7x_{32} + 4x_{33} + 3x_{34} \to \min .$$

The amount sent from and available at the warehouse W_i yields

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 200$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 300$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 450.$$

The amount sent to and required by the consumer C_j results in

$$x_{11} + x_{21} + x_{31} \geq 150$$
$$x_{12} + x_{22} + x_{32} \geq 300$$
$$x_{13} + x_{23} + x_{33} \geq 150$$
$$x_{14} + x_{24} + x_{34} \geq 200.$$

Negative amount from W_i to C_j is not allowed

$$x_{ij} \geq 0, \qquad i = 1, 2, 3 \text{ and } j = 1, \ldots, 4.$$

The total transportation cost is

$$z = 3x_{11} + 2x_{12} + 7x_{13} + x_{14} + x_{21} + 4x_{22} + \\ + 5x_{23} + 2x_{24} + 2x_{31} + 7x_{32} + 4x_{33} + 3x_{34} \to \min .$$

The amount sent from and available at the warehouse W_i yields

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 200$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 300$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 450.$$

The amount sent to and required by the consumer C_j results in

$$x_{11} + x_{21} + x_{31} \geq 150$$
$$x_{12} + x_{22} + x_{32} \geq 300$$
$$x_{13} + x_{23} + x_{33} \geq 150$$
$$x_{14} + x_{24} + x_{34} \geq 200.$$

Negative amount from W_i to C_j is not allowed

$$x_{ij} \geq 0, \qquad i = 1, 2, 3 \text{ and } j = 1, \ldots, 4.$$

The total transportation cost is

$$z = 3x_{11} + 2x_{12} + 7x_{13} + x_{14} + x_{21} + 4x_{22} + \\ + 5x_{23} + 2x_{24} + 2x_{31} + 7x_{32} + 4x_{33} + 3x_{34} \to \min .$$

The amount sent from and available at the warehouse W_i yields

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 200$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 300$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 450.$$

The amount sent to and required by the consumer C_j results in

$$x_{11} + x_{21} + x_{31} \geq 150$$
$$x_{12} + x_{22} + x_{32} \geq 300$$
$$x_{13} + x_{23} + x_{33} \geq 150$$
$$x_{14} + x_{24} + x_{34} \geq 200.$$

Negative amount from W_i to C_j is not allowed

$$x_{ij} \geq 0, \qquad i = 1, 2, 3 \text{ and } j = 1, \ldots, 4.$$

The total transportation cost is

$$z = 3x_{11} + 2x_{12} + 7x_{13} + x_{14} + x_{21} + 4x_{22} + \\ + 5x_{23} + 2x_{24} + 2x_{31} + 7x_{32} + 4x_{33} + 3x_{34} \to \min .$$

The amount sent from and available at the warehouse W_i yields

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 200$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 300$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 450.$$

The amount sent to and required by the consumer C_j results in

$$x_{11} + x_{21} + x_{31} \geq 150$$
$$x_{12} + x_{22} + x_{32} \geq 300$$
$$x_{13} + x_{23} + x_{33} \geq 150$$
$$x_{14} + x_{24} + x_{34} \geq 200.$$

Negative amount from W_i to C_j is not allowed

$$x_{ij} \geq 0, \qquad i = 1, 2, 3 \text{ and } j = 1, \ldots, 4.$$

Problem I

A dealer has 1500 EUR only for a purchase of rice and wheat. A bag of rice costs 150 EUR and a bag of wheat costs 120 EUR. He has a storage capacity of ten bags only and the dealer gets a profit of 11 EUR and 8 EUR per bag of rice and wheat, respectively. Formulate the problem of deciding how many bags of rice and wheat should dealer buy in order to get the maximum profit.

Problem II

Mr. Bob's bakery sells bagel and muffins. To bake a dozen bagels Bob needs 5 cups of flour, 2 eggs, and one cup of sugar. To bake a dozen muffins Bob needs 4 cups of flour, 4 eggs and two cups of sugar. Bob can sell bagels in 10 EUR/dozen and muffins in 12 EUR/dozen. Bob has 50 cups of flour, 30 eggs and 20 cups of sugar. Formulate the problem of deciding how many bagels and muffins should Bob bake in order to maximize his revenue.

Problem III

A small company produces two types of products bacon and cheese and sells them at a profit of 4 EUR/kg and 6 EUR/kg, respectively. A student is trying to decide on lowest cost diet that provides sufficient amount of proteins and fats. He knows that bacon contains 2 units of protein/kg, 5 units of fat/kg and cheese contains 2 units of protein/kg, 3 units of fat/kg. Moreover, for the proper diet student needs to consume 9 units of protein/day and 10 units of fat/day. Formulate the problem of deciding how much student should consume of food to meet the daily norm and the cost of food was minimal.

See you next week!

Hopefully.