ISS0031 Modeling and Identification

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Problem statement

Define a linear programming problem.

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A linear programming problem may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints.

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Define a convex set.

Let $S \neq \emptyset, S \subset \mathbb{R}^n$ and $x_1, x_2 \in S$.

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A set S in a vector space over $\mathbb{R}$ is called a convex set if the line segment joining any pair of points $x_1, x_2 \in S$ lies entirely in S.

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The set $[x_1, x_2] = \{x | x = \lambda x_1 + (1 - \lambda)x_2, 0 \leq \lambda \leq 1\}$ is called a line segment with the endpoints x_1, x_2.

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Determine two **non-negative** rational numbers such that their sum is maximum provided that their difference exceeds four. Furthermore, three times the first number plus the second should be less than or equal to 9. Formulate the problem as a linear programming problem and solve it using graphical method.

Let x_1 be the first and x_2 be the second number, then
Objective function:

$$z = x_1 + x_2 \to \max$$

Constraints:

$$x_1 - x_2 \geq 4$$
$$3x_1 + x_2 \leq 9$$
$$x_1 \geq 0, x_2 \geq 0$$
$$x_1, x_2 \in \mathbb{Q}$$

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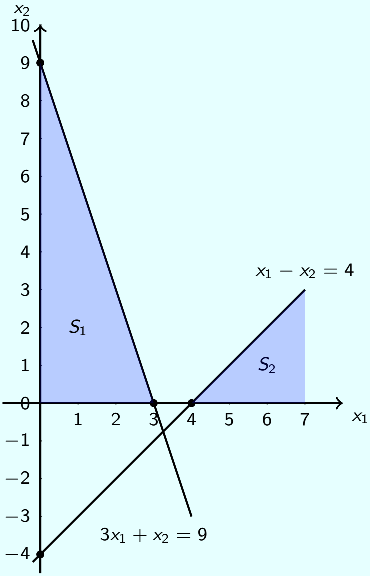

- $x_1 - x_2 = 4 \Rightarrow$ two points $(0, -4)$ and $(4, 0)$
- $3x_1 + x_2 = 9 \Rightarrow$ two points $(0, 9)$ and $(3, 0)$
- Since $S_1 \bigcap S_2 = \emptyset$ the problem has no solution.

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Mr. Bob's bakery sells bagel and muffins. To bake a dozen (i.e. 12 pieces) bagels Bob needs 5 cups of flour, 2 eggs, and one cup of sugar. To bake a dozen muffins Bob needs 4 cups of flour, 4 eggs and two cups of sugar. Bob can sell bagels in **10** EUR/dozen and muffins in **12** EUR/dozen. Bob has 50 cups of flour, 30 eggs and 20 cups of sugar. Formulate the problem of deciding how many bagels and muffins should Bob bake in order to maximize his revenue. Solve the problem by Simplex method.

Let x_1 be bagels and x_2 be muffins, then

$$z = \mathbf{10}x_1 + \mathbf{12}x_2 \to \max$$

$$5x_1 + 4x_2 \leq 50$$

$$2x_1 + 4x_2 \leq 30$$

$$1 \cdot x_1 + 2x_2 \leq 20$$

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Initial Simplex table:

x_1	x_2	s_1	s_2	s_3	z	b	variables
5	4	1	0	0	0	50	s_1
2	4	0	1	0	0	30	s_2
1	2	0	0	1	0	20	s_3
-10	-12	0	0	0	1	0	z

x_1	x_2	s_1	s_2	s_3	z	b	variables	ratio	transformations
5	4	1	0	0	0	50	s_1	$\frac{50}{4} = 12.5$	$R_1 - 4\tilde{R}_2$
2	**4**	0	1	0	0	30	s_2	$\frac{30}{4} = 7.5$	$\Rightarrow \tilde{R}_2 := \frac{R_2}{4}$
1	2	0	0	1	0	20	s_3	$\frac{20}{2} = 10$	$R_3 - 2\tilde{R}_2$
-10	-12	0	0	0	1	0	z		$R_4 + 12\tilde{R}_2$
3	0	1	-1	0	0	20	s_1	$\frac{20}{3} = 6.(6)$	$\Rightarrow \tilde{R}_1 := \frac{R_1}{3}$
$\frac{1}{2}$	1	0	$\frac{1}{4}$	0	0	7.5	x_2	15	$R_2 - \frac{1}{2}\tilde{R}_1$
0	0	0	-1	1	0	5	s_3		
-4	0	0	3	0	1	90	z		$R_4 + 4\tilde{R}_1$

x_1	x_2	s_1	s_2	s_3	z	b	variables
1	0	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{20}{3}$	x_1
0	1	$-\frac{1}{6}$	$\frac{5}{12}$	0	0	$\frac{25}{6}$	x_2
0	0	0	-1	1	0	5	s_3
0	0	4	$\frac{5}{3}$	0	1	$\frac{350}{3}$	z

In the above table we notice that there are **no** negative entries in the objective row.

Conclusion

The optimal solution has been found. Answer is $z_{\max}\left(\frac{20}{3}, \frac{25}{6}\right) = \frac{350}{3}$ or in pieces (1 dozen is 12 pieces) $z_{\max}(80, 50) = 1400$.