1 Processes

To solve control problem it is necessary to understand the process, we need to know what should be automated, no matter how advanced technique we can use!
(devices used to control are discussed in course ISS0060)

we discuss:

- description of the process (differential equations, transfer functions,...)
- simple processes (first and second order)
- process examples (thermal, chemical, level)
- process models

1.1 Description of the system

System: components(parts/units) and relationships between them.

there are 3 types of metter/energy changes in the system:

1. accumulation (gathering, storage)

   causes the transition processes

   capacity, lag/ mahtuvused/ емкостные

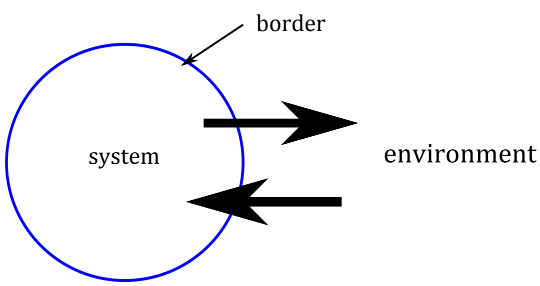
2. flows (movement)

   flows values are limited ($\neq \infty$)

   conductance/ juhtivus/ проводимость

3. loss

   energy $\rightarrow$ heat



	mass	energy
isolated systems	const.	const.
closed systems	const.	-
open systems	-	-

definitions are used

system...

- phases
- equilibrium

The purpose of the process description:
Is it possible to control? How to do that? What are the features?
System types:

- continuous (pidev / непрерывная) / discrete (diskreetne / дискретная)
- deterministic/ stochastic/ chaotic
- linear / nonlinear
- with lumped/distributed parameters
- SISO/ SIMO/ MIMO/ TITO
- stable (stabiilne / устойчивая)/ unstable (mittestabiilne / неустойчивая)
- controllable (juhitav / управляемая) / observable (jälgitav / наблюдаемая)/ robust (robustne / робастная)

Physical value (X) conservation laws [2]:

mass(m), load(q), energy (E), momentum(mv), rotation($J\omega$), etc.

Changes of the value can be described:

$$X_{accumulation} = X_{inflow} - X_{outflow} \pm X_{leakage/absorbtion}$$

change rate in the system	flow through the boarder	leakage/ absorbtion

This is process model.

Using the definition: flow $F = X/t$

$$\frac{\mathrm{d}X}{\mathrm{d}t} = F_s - F_v \pm F_t \tag{1}$$

This is the value X balance in one dimensional environment.

2 Process description techniques

- Differential equations $\dot{x} = F(x, u)$
- Transfer functions $H(s)$
- Frequency Response $H(j\omega)$
- Discrete time $H(z)$

Different views of reality, their relationships, process characteristics.

2.1 Differential Equation

Time domain

continuous physical processes

1. I/O representation $u(t), y(t)$

   $F(u, \dot{u}, \ldots, y, \dot{y}, \ldots) = 0$

   u^m, y^n, $n \geq m$, order (järk/ порядок)

   representation: operator $p = \mathrm{d}/\mathrm{d}t$

2. state-space representation (olekumudel/ пространство состояний)

   x - state variable

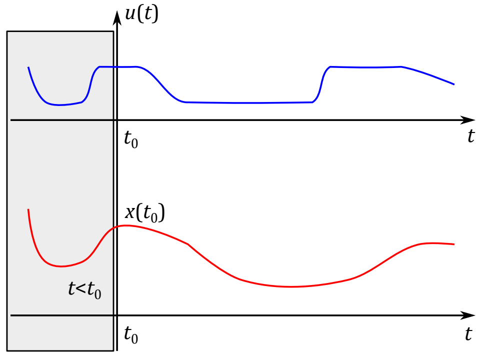
   $\dot{x} = f(x, u, p)$ p - parameters
   $y = g(x, u, p)$ $f(), g()$ - functions

   linear time invariant equation:

   $\dot{x} = Ax + Bu$ A - system matrix
   $y = Cx + Du$ B - inputs matrix
   C - outputs matrix
   D - feedthrough matrix (often $= 0$)

Initial parameters

Initial parameters $x(t_0)$ represent system memory of the past $t < t_0$
$x(t_0)$ and $u(t), t > t_0$ define the future behavior of the system $x(t)$.



Steady State

steady state (püsiolek / статика) working point where $\mathrm{d}x/\mathrm{d}t = 0$

$f(x, u) = 0$ associates inputs and outputs, often is nonlinear

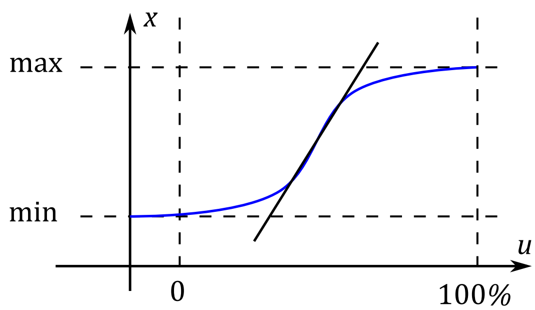


Figure 1: Process operating line

- exist limits $(\min, \max)$
- deviations from the linear dependencies

  $\pm 20\%$ - imperceptible

  $\pm 50\%$ - appreciable
- horizontal parts - input does not impact the output
- vertical parts - $k \gg$, oscillations

Process steady state is equally important as process dynamics.

Choice of the working point (u_0, x_0).
Linearization of the equation $\dfrac{\mathrm{d}x}{\mathrm{d}t} = f(x,u)$ at the working point (u_0, x_0)

where
$f(\Delta u, \Delta x) = 0$ describes the deviation variables $\Delta u, \Delta x$ at the working point $(u = u_0 + \Delta u, x = x_0 + \Delta x)$;
$f(x,u)$ - continuous and differentiable.

$$\Delta \dot{x} = \frac{\mathrm{d}f}{\mathrm{d}x} \cdot \Delta x + \frac{\mathrm{d}f}{\mathrm{d}u} \cdot \Delta u$$

describes the system dynamics

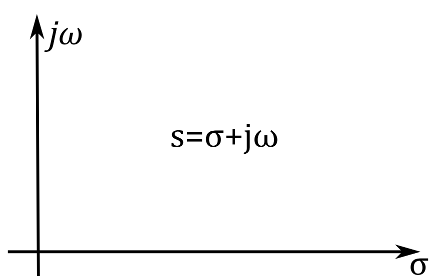
$\Delta u, \Delta x$ - are the new state variables
working point is $\Delta x = 0$

2.2 Transfer function

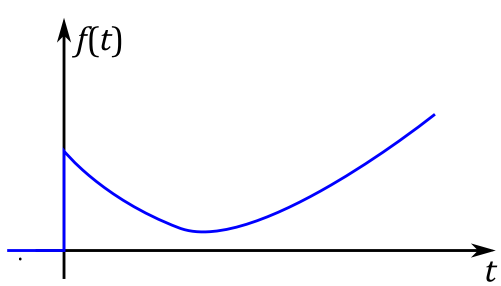
solutions in the frequency domain s domain
Laplace Transform $f(t)$

$$f(t) \leftrightarrow F(s) \quad \text{variable } s(1/s) \quad \text{changing rate}$$

$$F(s) = \int_0^\infty f(t) \cdot e^{-st} \mathrm{d}t \tag{2}$$



$$F(s) = L[f(t)]$$
$$f(t) = L^{-1}[F(s)]$$



function $f(t)$

$$f(t) = \begin{cases} 0, & \text{t<0;} \\ f(t), & \text{t>0.} \end{cases}$$

If object is stable we can find values of the time domain functions at two extremes $t = 0$ and $t = \infty$, without inverse transform.

Initial-value theorem

$$\lim_{s\to\infty}[sF(s)] = \lim_{t\to} f(t) \tag{3}$$

Final-value Theorem

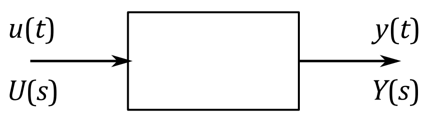
$$\lim_{s\to 0}[sF(s)] = \lim_{t\to\infty} f(t) \tag{4}$$

Differential equation $\rightarrow$ algebraic equation

zero initial conditions give simplified results

In control we use final-value theorem quite often [1].

Transfer function model $H(s), G(s), W(s), \ldots$



output depends on the input, zero initial values $Y(s) = H(s) \cdot U(s)$,

$$H(s) = k \cdot \frac{s^m + \cdots + b_0}{s^n + \cdots + a_0},$$

steady state transfer: $H(s)_{s=0} \ldots y(\infty)$

$$H(s) = k \cdot \frac{(s - z_1)(s - z_2)\ldots(s - z_m)}{(s - p_1)(s - p_2)\ldots(s - p_n)},$$

zeros (m) and poles (n)

If poles are real:

$$H(s) = K \cdot \frac{(T_{z_1} + 1)\ldots}{(T_{p_1} + 1)(T_{p_2} + 1)\ldots}$$

Realizability of the transfer function defines ratio $n : m$

strictly proper realizable if $n > m$

$\lim_{s\to\infty} H(s) = 0$, in practice $H(\omega \to \infty) = 0$

semi proper, biproper $\ldots = /0$

improper not realizable if $n < m$

$\lim_{s\to\infty} H(s) = \infty$

Poles of the system p_i are state matrix A eigenvalues

- $[A, B, C, D] \rightarrow H = C \cdot (sI - A)^{-1} \cdot B + D$
  during the transfer function calculation zeros and poles can withdraw
- backward $[A\ B\ C\ D] \leftarrow$ is not uniquely defined, depend on the state choice

  H can contain less information than $[A\ B\ C\ D]$

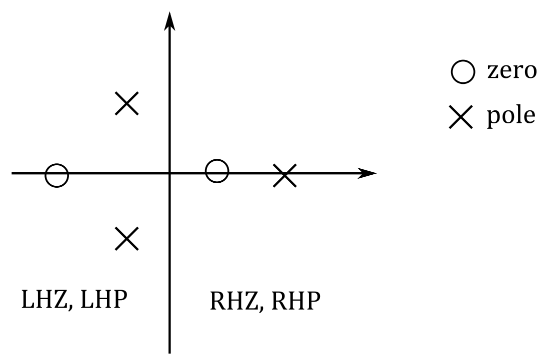


Figure 2: Designation of the plane $s = \sigma + j\omega$

Stable system: $Re(p) < 0$
Unstable system : $Re(p) > 0$
The poles closer to the origin are **dominant**!

Why frequency domain (s domain) with transfer function $H(s)$?

- simpler than in time domain
- fully describes dynamics
- property of the system, does not depend on the inputs
- valid for the initial values

Characteristics

1. step $g(t)$

   $u(t) = 1(t)$

   $U(s) = 1 \cdot 1/s$

2. impulse response h(t)

$u(t) = \delta(t)$

$U(s) = 1$

3. ramp r(t)

$$u(t) = \sigma(t) = \begin{cases} 0, & \text{t<0;} \\ t, & \text{t>0.} \end{cases}$$

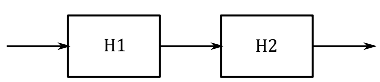
$U(s) = 1/s^2$

Figure 3: step, impulse, ramp signals

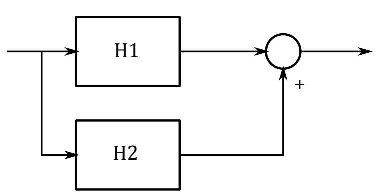
Those co-called "singular functions" d/d$t = \infty$ are suitable for system tests. Extreme modes: if works here, then works well with other signals as well.

Block-diagram reduction

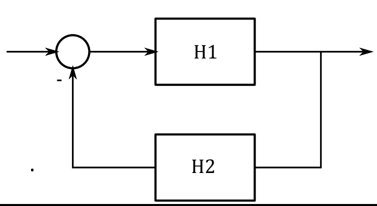
Block-diagrams illustrate a cause-and-effect relationship. Blocks are used to represent transfer functions and lines for indirections information transmission.



$H = H1 \cdot H2$



$H = H1 + H2$



$$H = \frac{H1}{1 + H1 \cdot H2} = \frac{\text{"}direct\text{"}}{1 + \text{"}loop\text{"}}$$

Using transfer functions

- get results without resolving differential equations;

  stability, final-values ($t = \infty$)

- impacts are clearly shown;

  differences and similarities

- suitable for linear SISO systems.

2.3 Frequency Response

description of the frequency domain

If sinusoidal input is imposed and frequency response is measured the dynamic behavior of the system can be studied. Bode and Nyquist plots are graphical representations of functional dependance of magnitude and phase on frequency.

$H(j\omega) = y/u = M \cdot e^{j\phi}$

1. magnitude and phase frequency characteristic $M(\omega)$
2. phase frequency characteristic $\phi(\omega)$
3. complex frequency characteristic $M(j\omega)$

steady-state: $H(\omega = 0)$
relationship with transfer function: $H(j\omega) = H(s)_{s=j\omega}$

log scale for the magnitude

any pole on the frequency $\omega > \omega p$ causes

- change of the magnitude $-20\ dB/dec$
- phase change $-\pi/2$

Bibliography

[1] Pao C. Chau, *Process Control: A First Course with MATLAB*. Cambridge University Press, 2002.

[2] D. E. Seborg, and T. F. Edgar, and D. A. Mellichamp, and F. J. Doyle III *Process Dynamics and Control*. John Wiley & Sons, 2011.