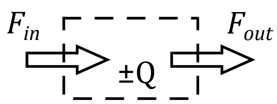
Process examples

1 Thermal processes



- fix a border of the system (part of the environment)
- write down the system's energy equilibrium point

$$Q_{accumulation} = Q_{in} - Q_{out} \pm Q_{occurs/absorbs}$$

$$\frac{\mathrm{d}}{\mathrm{d}t} Q_{ac} = F_{in} - F_{out} \qquad (1)$$

changing rate in some environment — $\pm$ flows through the border $F = \mathrm{d}Q/\mathrm{d}t$

Key concepts:

the **quantity of heat** Q (energy) Units: $J = W \cdots, kW \cdot h, MW \cdot h$

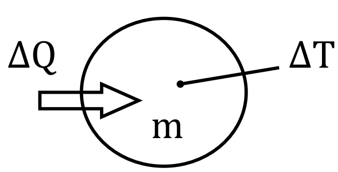
heat flow $F = \mathrm{d}Q/\mathrm{d}t$ (power) Units: W, kW

temperature indicator: T, t, θ Units: $^{\circ}C, K$

Processes: heat accumulation, heat transfer

1.1 Heat accumulation

Temperature rise $\Delta Q \rightarrow \Delta T$



$$\Delta Q = c \cdot m \cdot \Delta T$$

$$\left(\sum c_i \cdot m_i\right) \cdot \Delta T$$

heat capacity c Heat Capacity, Specific Heat

property of matter Units: $kJ/(kg \cdot^\circ C)$

water	4.18
iron	0.44
fuel oil	2.5
ice, water vapor	2.0
air	1
Al	0.897

1.2 Heat Transfer

mass exchange, heat conduction, convection, radiation

1. Occurrence / absorption

   - heating with power P, burning

     calorific value (heat of combustion), MJ/kg

     heating oils 42, H_2 121; C 32;

   - phase transfers (solid-liquid-gas)

     melting of ice = $334kJ/kg$ ($T = 0°C$)

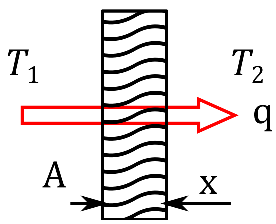
     boiling water = $2256kJ/kg$ ($T = 100°C$)

2. Mass exchange: given quantity m of substance, at temperature T

   $Q = c \cdot m \cdot T,\ F = Q/t = c \cdot q \cdot T$ - heat flow

3. Thermal conduction

temperature difference $T_1 - T_2 \rightarrow$ heat flow q



$$q = -\lambda \cdot \mathrm{d}t/\mathrm{d}x \quad W/m^2 \tag{2}$$

heat flow q per surface $A = 1\ m^2$
is $\sim$ temp. gradient

Thermal conductivity λ - property of the material,
units are $W/(m \cdot K)$
Cu 390, Al 210, Fe 50, steel 16 , concrete 1.55, water 0.61 , ice 2.2, wood 0.11, silicate cotton 0.04, leather 0.017, fat 0.21, air 0.024, etc.

With a fixed thickness x $(areaA = 1\ m^2)$ parameters:

$$q = \alpha \cdot \Delta T = \Delta T/R, \tag{3}$$

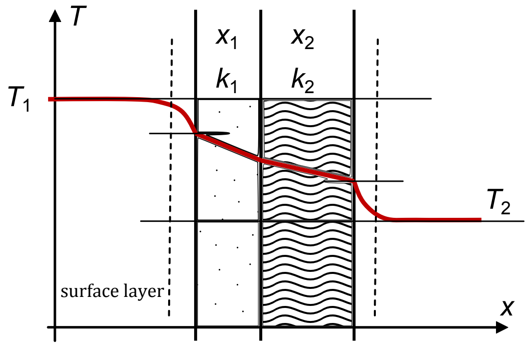
where α - heat transfer coefficient $\alpha = \lambda/x \quad W/(m^2 \cdot K)$
R - thermal resistance $R = x/\lambda \quad m2 \cdot K/W$

Example:
Calculate: stainless steel wall thickness of 3 mm, $R =?$

With multi-layer structure thermal resistances
$\sum R_i = x_1/\lambda_1 + x_2/\lambda_2 + \ldots$

4. Convection - heat transfer though gas and liquid environment (2 environments!)

   1 m^2 of the surface layer has the heat transfer coefficient α (thermal resistance $1/\alpha$) units: $W/(m^2 \cdot K)$



gas, natural convection 3-25

motionless air 2.7

fluid, natural convection from 30 to 60

boiling water 4000 to 15000

vapor condensation from 10000 to 20000

The total heat transfer consists of components
$convection + heat\ transfer + convection$ with a thermal resistance
$R_h = 1/\alpha_1 + x/\lambda + 1/\alpha_2 = 1/\alpha$
and heat transfer coefficient α

gas - gas	$10 \ldots 35$
plastic window	~ 1
water - water	$850 \ldots 1700$
oil - oil	$100 \ldots 300$
steam - air	$35 \ldots 90$
steam - oil	$280 \ldots 2300$
steam - boiling water	$1700 \ldots 4500$

The total heat transfer (the whole surface A)

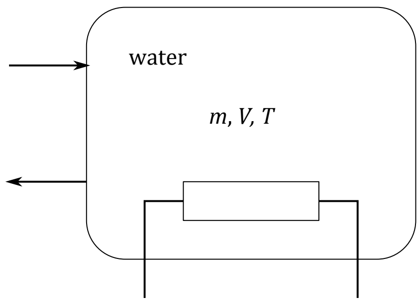
$$q = \alpha \cdot A \cdot \Delta T = A \cdot \Delta T / R_h \tag{4}$$

Calculate: stainless steel wall thickness of 3 mm, one side is the water ($T = 100°C$) on the second side is the air ($T = 20°C$), $R =$?

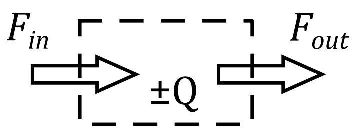
5. Radiation transfer $q = \alpha \cdot A \cdot T^4$

Calculate: How quickly cools teapot $V = 1\ L$ with convention and radiation ($\alpha = 5.67 \cdot 10^{-8} \quad W/(m^2 \cdot K^4)$)?

Example 1 *Water boiler*



m - water mass (50 l, 50 kg) = const tank is full;
T - water temperature, T_e - environment temperature;
P - heater power (3 kW).



General rule:
$\frac{dQ}{dt} = F_{in} - F_{out}$
temperature $T(t, P, m, T_e, \ldots)$

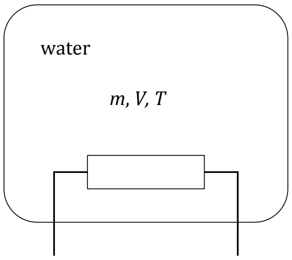
- heat in-flow (heat source) $F_{in} = P$
- heat out-flow (cooling due to environment) $F_{out_e} = K \cdot (T - T_e)$
- heat out-flow (running water) - mass exchange with quantity $q = \mathrm{d}m/\mathrm{d}t = q_{in} = q_{out}$

  $F_{out_f} = c \cdot q \cdot (T - T_i)$
- quantity of heat in the boiler $Q = c \cdot m \cdot T$

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = F_{in} - F_{out_e} - F_{out_f}$$

Consider specific cases:

1. There is no flow and cooling into environment $F_{out_e} = F_{out_f} = 0$



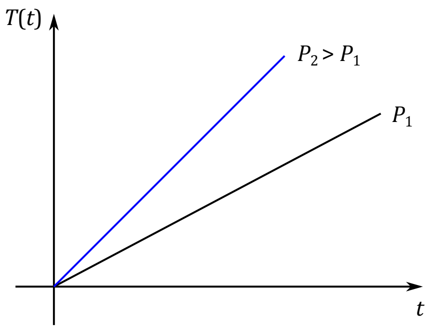
Water is heated

$$\frac{\mathrm{d}(c \cdot m \cdot T)}{\mathrm{d}t} = P \qquad cm\frac{\mathrm{d}T}{\mathrm{d}t} = P \text{ - integrator}$$

$$P \to T \text{ transfer } T(s) = \frac{1}{s}\frac{1}{cm}P(s)$$

steady state (statics) $T(P)$?

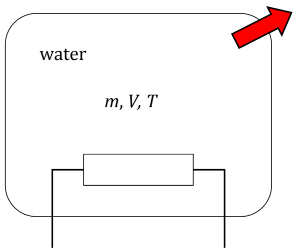
step response (heating)



- what is the initial temperature $T(0)$?
- what happens if heating is stopped $P = 0$?

Calculate: when the water temperature reaches $+40°C$

2. There is no flow, the cooling into environment is present $F_{out_f} = 0$



differential equation $K \neq 0$

$$\frac{\mathrm{d}(c \cdot m \cdot T)}{\mathrm{d}t} = P - K \cdot (T - T_e)$$

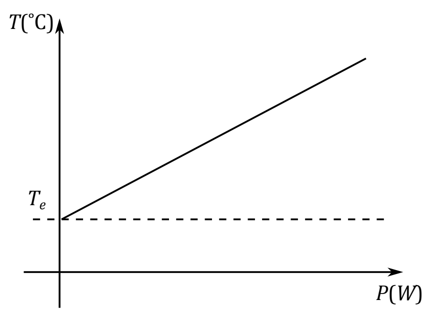
$$K \cdot T + cm\frac{\mathrm{d}T}{\mathrm{d}t} = P + K \cdot T_e$$

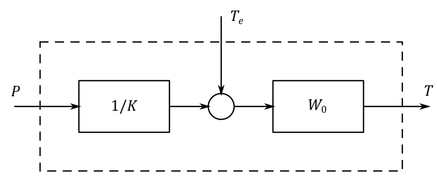
transfer $T(s) = \dfrac{1/K \cdot P(s) + T_e(s)}{1 + \dfrac{c \cdot m}{K} \cdot s}$,

time constant $\tau = cm/K$

steady state $\mathrm{d}T/\mathrm{d}t = 0; s = 0 \rightarrow$ final temp. $T(P, T_e)$

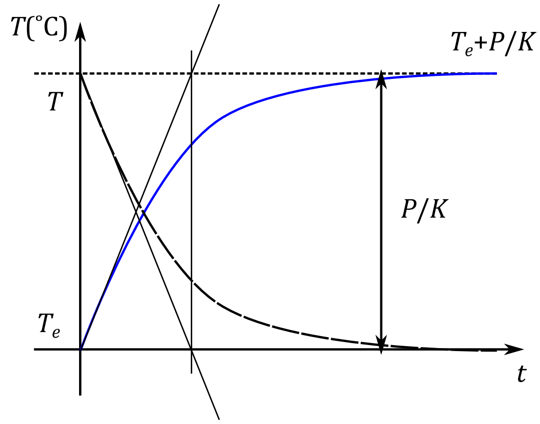
$T - T_e = \dfrac{1}{K} \cdot P$ the step response:



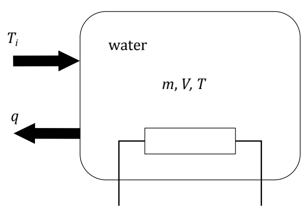


- equation is valid if T is lower than boiling temperature!
- cooling: $P = 0, T(t = 0) \neq T_e$

Calculate: when the heating is OFF ($P = 0$), with temperature $+60\ ^\circ C$ boiler is cooling about $5\ ^\circ C$ $15\ min$. Find gain K.



3. Running water is heated



good isolation $K = 0$

$$\frac{\mathrm{d}(c \cdot m \cdot T)}{\mathrm{d}t} = P - c \cdot q(T - T_i)$$

$$cqT\frac{\mathrm{d}T}{\mathrm{d}t} = P + cqT_i \text{ transfer } T(s) = \frac{1/qc \cdot P + T_i}{1 + (m/q) \cdot s},$$

time constant $\tau = m/q$

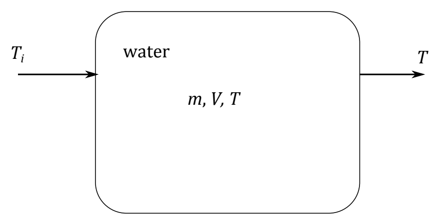
special cases:

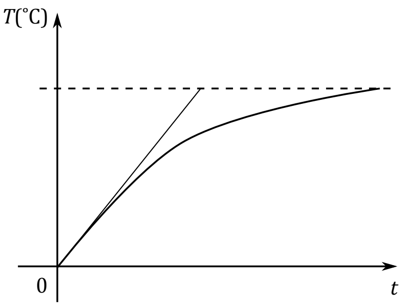
- water is running through the tank, no heating $P = 0,\ K = 0$

  is the filter for the in-flow water temperature T_i

$$T = \frac{T_i}{1 + (m/q) \cdot s}$$

- if several tanks are in series then $W = W_1 \cdot W_2 \cdot \ldots$
- if several in-flows then $(q_1, T_1) + (q_2, T_2) \rightarrow (q_1 + q_2, T)$

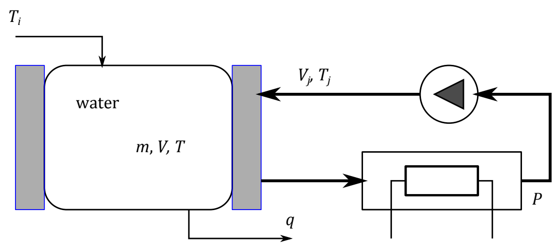




mix of the hot and cold water

What is happening if $V = 0$?

Example 2 *Jacketed water heater*



temperature sensitive chemical processes, jacketed bio-processes (water or oil), heating P

two environments: the tank (V, T, q flow-through), jacket (V_j, T_j)

both with uniform temperature throughout the volume;

heat transfer by thermal conductivity from jacket to the tank;

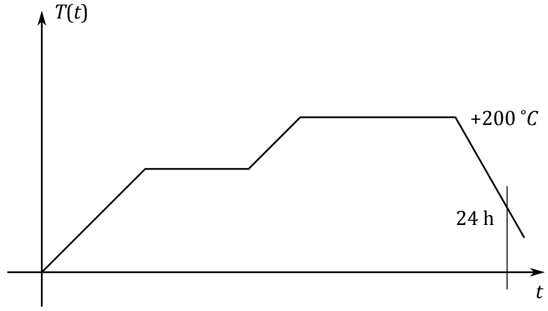
heat losses to the environment are not taken into account;

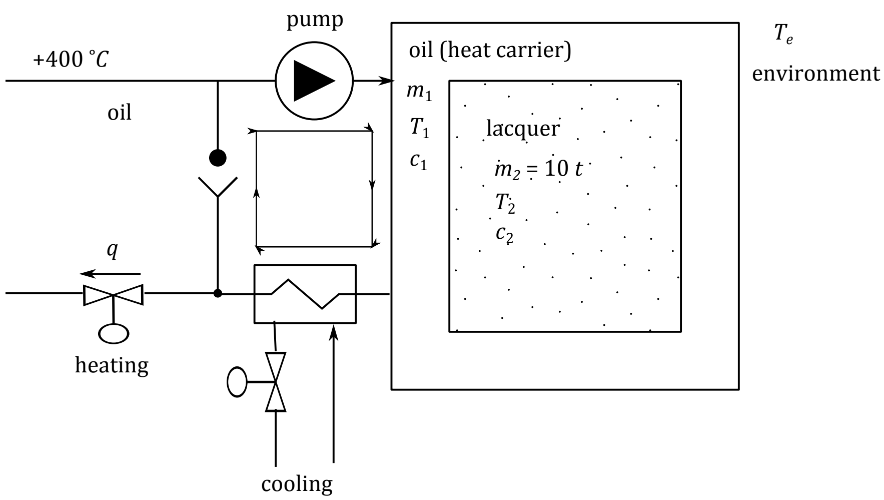
process in the tank does not absorb or release heat.

$$\mathrm{d}/\mathrm{d}t\ Q = F_{in} - F_{out}; \qquad Q = cmT; \qquad m = \rho V$$

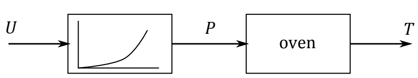
Reactor of the chemical process `AS Sadolin (Rapla)`

Task: programmable control of the temperature
equipment:





Example 3 *Oven*



nonlinear steady state characteristic

$$T - T_e \equiv P,\ P = c \cdot U^2$$

$$T = T_e + c \cdot U^2$$

Model: nonlinear steady state + linear dynamics

Control oven power, not voltage U! (then linear)

Bibliography

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[3] F. G. Shinskey, *Process Control Systems: application, design, and tuning*. McGraw-Hill, 1996.