Process examples

1 Level Control

Is the purpose to gather material?

1. stocks (the necessary quantity);

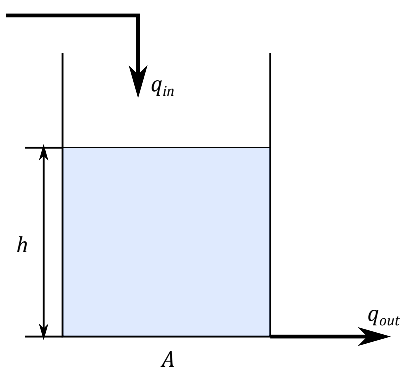
   intermediate warehouses between continuous and batch production,
   min stock ($, safety) ⇔ sufficiency stock.

2. smoothing the disturbances of the continuous processes (averaging);

3. separation, gathering [oil, mud, air, gas, steam]

   separator, deaerator

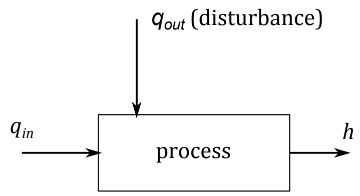
1.1 Description of the process



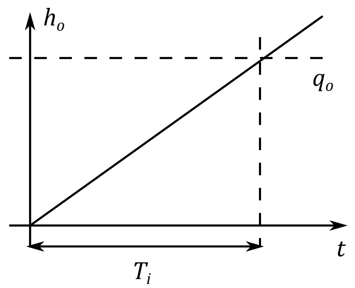
space: container, inventory, tank, vessel

Volume V;
Flow: q, Q, F;
Units: [amount of substance /time units]
$m^3/h, L/min, kg/s, \ldots$
Amount: $V = \int q \cdot \mathrm{d}t$

law of conservation of mass → volume conservation, ($M = \rho \cdot V, \rho = const$)



$$\Delta V = V_{in} - V_{out}$$
$$\frac{\mathrm{d}V}{\mathrm{d}t} = q_{in} - q_{out},\ V = \int (q_{in} - q_{out}) \cdot \mathrm{d}t$$



in case of vertical cylindrical tank

$V = A \cdot h$

$h = \dfrac{1}{A} \int (q_{in} - q_{out}) \cdot \mathrm{d}t$

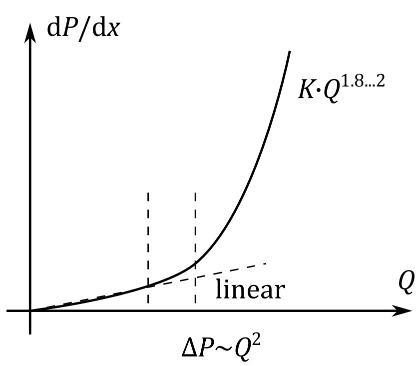
integrator $W(s) = K/s$

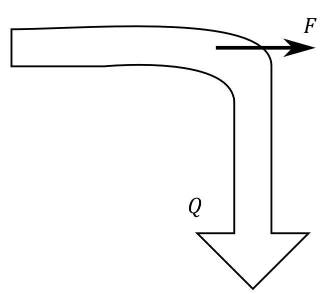
Normalized input/output values $h' = \dfrac{h}{h_o} \cdot 100\%, \quad q' = \dfrac{q}{q_o} \cdot 100\%$

1.2 Liquids

- steady state (statics)
  - liquid conserves the volume $V = const$
  - pressure of liquids $\mathrm{d}P/\mathrm{d}h = -\rho g, \quad P = \rho g h$
  - force to the area A is $F = PA$
- laminar or turbulent flow

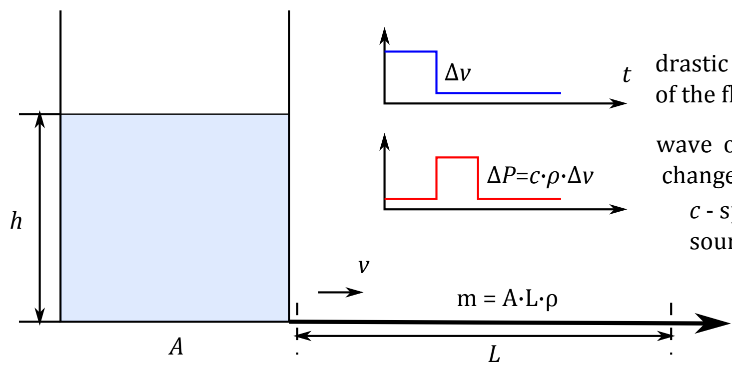
  force is caused by a change in flow the direction





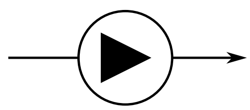
$F = m \cdot a = m \cdot \Delta V / \Delta t$ (pulse)

- Hydraulic shot



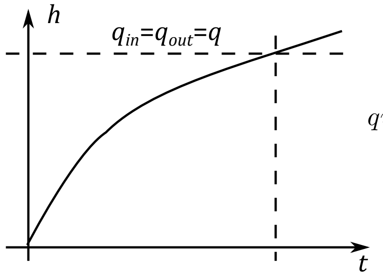
On what do the inflow and outflow depend?

1. pumps



$q_{in},\ q_{out} \neq f(h),\ q \approx const$

2. outflow depends on pressure (level), free outflow



$qv \equiv \sqrt{h}$ - nonlinear!

$$A \cdot \frac{\mathrm{d}h}{\mathrm{d}t} = q_{in} - K_1 \cdot \sqrt{h}$$

steady state: $q_0 = K_1 \cdot \sqrt{h_0}$

Can be

✓ solved numerically,

✓ linearized at operating point (q_0, h_0)

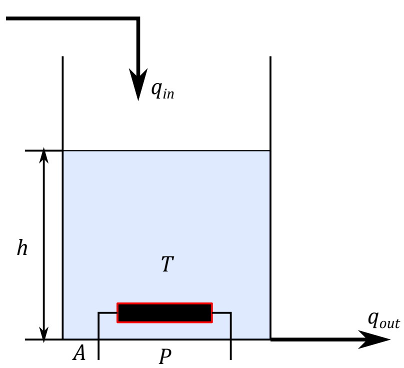
$$F(x_0 + \Delta x) \approx F(x_0) + \frac{\mathrm{d}F}{\mathrm{d}x}\Delta x + \ldots \tag{1}$$

$$W(s) = \frac{\sqrt{h_0}/K_1}{1 + (\frac{A\cdot\sqrt{h_0}}{K_1}) \cdot s} = \frac{K}{1 + T \cdot s} \tag{2}$$

Values of the gain K and time constant T depend on operation point h_0!

2 Level control + thermal process

The quantity of the heated liquid $V = A \cdot h$ and temperature T are changing



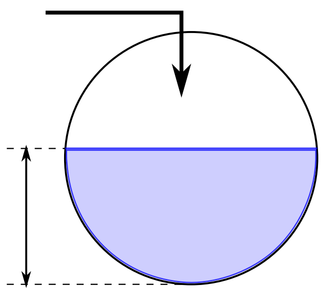
mass:
$\mathrm{d}/\mathrm{d}t\ (\rho A h) = \rho \cdot q_{in} - \rho \cdot q_{out}$
$\mathrm{d}h/\mathrm{d}t = F_1(q_{in}, q_{out})$
energy:
$\mathrm{d}/\mathrm{d}t(c \cdot \rho A h \cdot T) = P + c \cdot q_{in} T_i - c \cdot q_{out} T$, where $h(t), T(t)$
and $\mathrm{d}/\mathrm{d}t\ () = c \cdot \rho A(T \cdot \mathrm{d}h/\mathrm{d}t + h \cdot \mathrm{d}T/\mathrm{d}t) = \ldots$
$\mathrm{d}T/\mathrm{d}t = F_2(h, T, q_{in}, q_{out}, P, T_i)$

This process in nonlinear, linearize at operating point h_0, T_0

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} h \\ T \end{pmatrix} = \begin{pmatrix} * & * \\ * & * \end{pmatrix} \cdot \begin{pmatrix} h \\ T \end{pmatrix} + \begin{pmatrix} * & * & * \\ * & * & * \end{pmatrix} \cdot \begin{pmatrix} P \\ q_{in} \\ q_{out} \end{pmatrix}$$

2.1 Notes

- the relationship between level and volume depends on the shape of the container

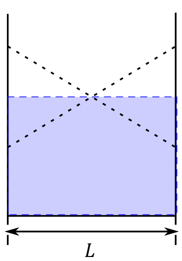


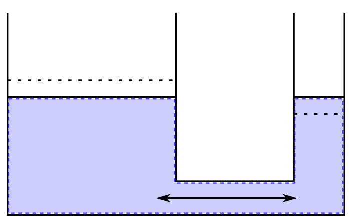
spherical shape: $V = \pi \cdot \frac{h^2 \cdot (3r - h)}{3}$

cylindrical: $V = \pi D^2/4 \cdot h$

- waves and oscillations in the tank [2]

  the mass movement (pot. and kin. energy) develops when some force creates a difference in levels between two points. Difference accelerates a flow. on reaching wall much momentum remains, so flow is reversed and action repeated.



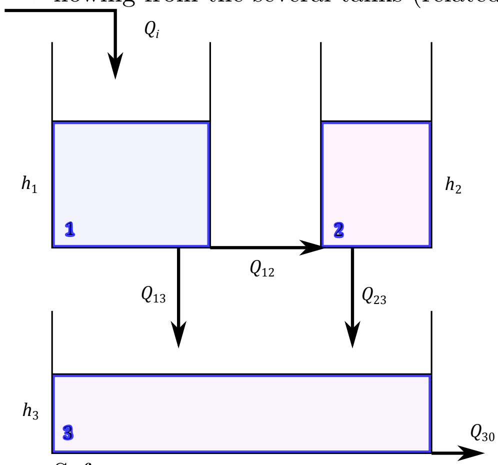


the period of the wave

resonance: $T = 2\pi\sqrt{\frac{L}{2g}}$,

where L is the distance of opposite walls.

- flowing from the several tanks (related vessels)



$$A_i \frac{\mathrm{d}h_i}{\mathrm{d}t} = Q_{in} - Q_{out}$$
$$Q_{ik} = F(h_i, h_k)$$

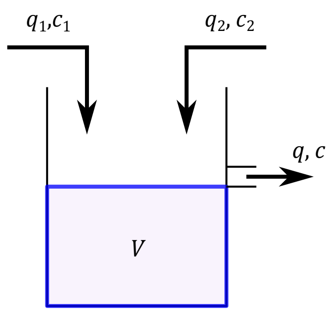
- Safety
  - combustible, explosive, toxic, fragrant liquids
  - pressurized containers
  - overflow causes pollution

2.2 Heating and ventilation systems

HVAC - Heating Ventilating and Air Conditioning
90% of the time a person is indoors!
Objective: The desired temperature and air quality
min energy expenses

3 Mixing processes

Substance (X) is included in the total quantity of the substance.
concentration $c = \frac{m_x}{\sum m}[\frac{kg, L, mole}{kg, m^3}]$ $c = (0 \ldots 1)$ or $(0\% \ldots 100\%)$
$c \geq 0$ a non-negative!
mass balance: conservation of mass $m_{in} - m_{out} = m_{accumulation}$
$m = \rho \cdot V,\ \rho = const \quad \rightarrow \quad \sum V_{x_{in}} - \sum V_{x_{out}} \approx V_{x_{accum}}.$



two inflows q_1, q_2
outflow: Over-flow ($V = const$)
$q = q_1 + q_2$
regardless of the inflows

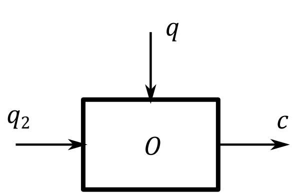
$$V = q \cdot \Delta t,\ V_x = c \cdot V$$

$$V_{x_{in}} - V_{x_{out}} = \Delta V_x$$

$$c_1 \cdot q_1 \cdot \Delta t + c_2 \cdot q_2 \cdot \Delta t - c \cdot (q_1 + q_2) \cdot \Delta t = \Delta c \cdot V$$

$$V \cdot \mathrm{d}c/\mathrm{d}t + c \cdot (q_1 + q_2) = c_1 \cdot q_1 + c_2 \cdot q_2 \tag{3}$$

Steady state: $c = \ldots,$
Transfers: exponential time constant $\tau = v/(q_1 + q_2)$
Gain: Input $q_2, c_2 = 0$



$$\Delta = \left(\frac{-c_1}{q}\right)\Delta q_2$$

4 Mechanical processes

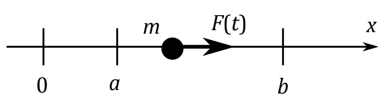
(solid movement)

1. Mass point Isaac Newton's formalism The acceleration a of a body is parallel and directly proportional to the net force F acting on the body, is in the direction of the net force, and is inversely proportional to the mass m of the body, i.e., $\sum F = m\ddot{x}$.

   - inertial system x
   - restrictions to the forces $|F| < F_{\max}$

     to the movements: space, range $x_1 \ldots x_2$

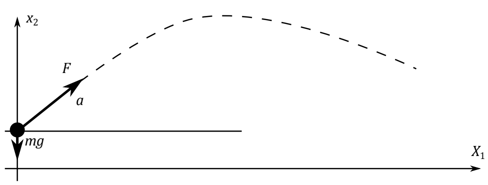
   - one dimensional



$$m \cdot \ddot{x} = F$$

calculate: $F = 10\ N,\ \Delta t = 20\ ms,\ m = 0.5\ kg,\ x, v =?$ state space?

   - two dimensional (in a vertical gravitation field)



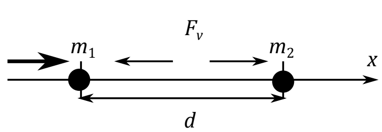
$$m \cdot \ddot{x_1} = F \cdot \cos\alpha$$
$$m \cdot \ddot{x_2} = F \cdot \sin\alpha - mg$$

2. several (n) points (masses)

   restrictions distance $x_1^2 + x_2^2 + x_3^2 = const$

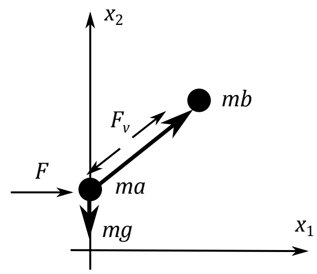
   $\sum_{j=1}^{N}(m_j\ddot{r_j} - F_j) = \sum \vec{F_j}$

   - two masses in one dimensional space



$$m_1 \cdot \ddot{x}_1 = F - F_v$$
$$m_2 \cdot \ddot{x}_2 = F_v$$
$$x_2 - x_1 = d$$

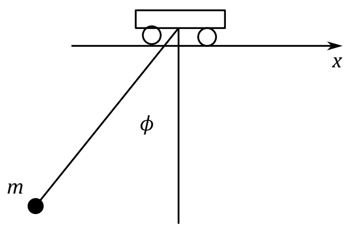
   - Two mass points in two-dimensional space



3. several moving bodies

   Lagrangian formalism

   $L = T - \Pi$ /difference between kinetic and potential energy /



$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\frac{\partial L}{\partial \dot{q}}\Big) = \frac{\partial L}{\partial q}$$

$$\text{state: } \begin{pmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{pmatrix}$$

as a result models are used to control

5 Chemical processes

The kinetics of chemical reactions, flow; concentration of a substance $[A]$ (mol/l);

Example: The reaction $A + B \leftrightarrow C$ equation

$$\frac{\mathrm{d}}{\mathrm{d}t}[B] = -k \cdot [A] \cdot [B]$$

- speed of the reaction k directly depends on the temperature

  $k = k_0 \cdot e^{-E/RT}$

  nonlinear and often unstable processes

- release or absorbtion of the heat

Chemical Processes: continuous or batch processes

- polymerization

- biotechnology

Temperature stabilization and control task for nonlinear, unstable process.

Bibliography

[1] William L. Luyben, *Process Modeling, Simulation, and Control for Chemical Engineers*. McGraw-Hill, 1990.

[2] F. G. Shinskey, *Process Control Systems: application, design, and tuning*. McGraw-Hill, 1996.