1 Process models

`the desire to describe reality`

Model of the process, model simplification, identification.

- Replaces the original;
- Maintains the essential characteristics, describes the process;
- One process can be described by the several models.

Model is used on purpose:

- Similarity of appearance - material model;
- Understanding of the process, predicting of the behavior to change the process (properties) in desired direction

  efficiency, safety, control, etc.

How to develop a model?

- Experimental data / empirical model / so called "Black box"
- Computationally/ functional model / where

  based on the components and the laws of nature: differential equations, state-space representation, and transfer function

- Combined method, so called "gray box".

Type of the model will depend on assumptions initially made to define the system. In general, the more assumptions we make, the simpler the structure of the model will be[3].

How to check a model?
The model is always approximate, the model must be

- "Accurate enough" to describe the ;

- "Simple" to calculate.

Model should be specified or simplified if needed.
The model is specified if its features can be changed in the area that interests us (closed system behavior).
The model is simplified if its parameters are insignificant.

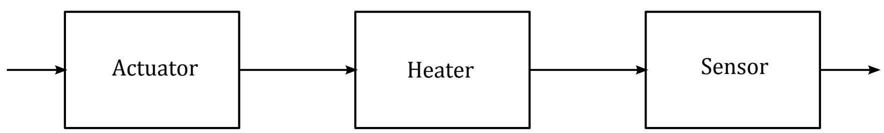
Our goal: We are looking for the models that describe what is important for the process in a closed system. These models allow you to choose the controller, the descriptive reality as similar to the behavior of a closed system, will provide answers to key questions:

When the system is stable?

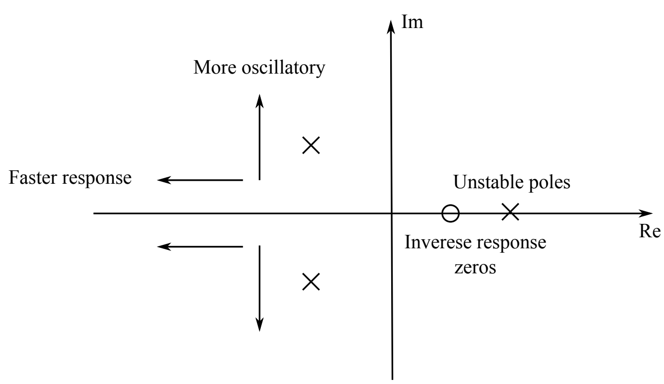
What are the characteristics of the system?

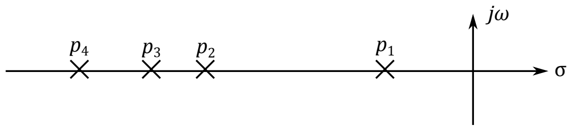
Example 1 *Heater*

Model of the system ← models of the components



$$W(s) = \frac{K_1}{1 + T_1 \cdot s} \cdot \frac{K_2}{(1 + T_2 \cdot s)(1 + T_3 \cdot s)} \cdot \frac{K_3}{1 + T_4 \cdot s}$$





Time constants: T_1, T_2, T_3, T_4
(poles $p = -1/T$)

Do we need so accurate (complex) model for the control?
Simplification: the model reduction

- State-space representation A, B, C, D `non-critical states?`
- Transfer function $W(s)$ `not important poles?`

1. Considering the dominant pole p_1 ($\min|p| - \max T$),

   Do not consider others, **First Order** model.

   Too simple to describe the system with required accuracy, can be used to describe the components.

2. Take into account two important poles p_1, p_2

   Do not consider others, **Second Order** model.

   more accurate model, not exact, there are better models.

   In real word where are not purely first or second order systems, there are additional precesses which impacts should be considered.

3. Several distant poles $p_1, p_2, \ldots$ with time constants $T_1, T_2, \ldots$

   Time constants are summarized and provided as delay $W(s) = e^{-s\sum T_i}$

   Important dynamical feature of the object provided as time constant, insignificant - as delays (models **FOPDT**, **SOPDT**)

1.1 FOPDT model

First Order Plus Dead Time

$$W(s) = \frac{K}{1 + T \cdot s} e^{-\tau \cdot s} \tag{1}$$

where
K_p - `process gain`: the ultimate value of the response (new steady-state) for a unit step change in the input.
T - `Time constant`: measure of time needed for the process to adjust to a change in the input.
τ - `Delay`: the time at which output of the system begins to change minus the time at which the input step change was made [3].

- Three parameters: K, T, τ simple but moderately complex.

- Describes the dynamics of the system with sufficient accuracy

  controllers work on this basis.
- Easily obtained with simplification of the complex models;
- Easy to identify.

The problem: if the highest insignificant time constant (T_2) is close to the important time constant (T_1)

Example 2 *Approximation with FOPDT*

$$\frac{1}{1+T_p s} \approx e^{-T_p s}; \tag{2}$$

$$1 - T_z s \approx e^{-T_z s} \tag{3}$$

If important T_1 is the largest time constant $T_1 > T_2 > T_3$, the third order system can be approximated as follows:

$$\begin{aligned}\frac{K_p}{(1+T_1 s)(1+T_2 s)(1+T_3 s)} &\approx \frac{K_p}{1+T_1 s}\cdot\frac{1}{1+T_2 s}\cdot\frac{1}{1+T_3 s}\\ &\approx \frac{K_p}{(1+T_1 s)}\cdot e^{-T_2 s}\cdot e^{-T_3 s}\\ &= \frac{K_p}{(1+T_1 s)} e^{-(T_2+T_3)s}\end{aligned}$$

Example 3 *Skogestad method [6]*

The largest neglected time constant should be divided between the smallest retained time constant and the time delay:
for the first order model:

$$T_{10} = T_1 + T_2/2, \quad \tau = \tau_0 + T_2/2 + \sum_{i\geq 3} T_{p_i} + \sum_j T_{n_j} \tag{4}$$

for the second order model:

$$T_{10} = T_1, \quad T_{20} = T_2 + T_3/2, \quad \tau = \tau_0 + T_3/2 + \sum_{i\geq 4} T_{p_i} + \sum_j T_{n_j} \tag{5}$$

if we have positive numerator against neighboring denominator

$$\frac{1+T_z s}{1+T_p s} \approx \begin{cases} T_z/T_p & \text{for } T_z \geq T_p \geq \tau \\ T_z/\tau & \text{for } T_z \geq \tau \geq T_p \\ 1 & \text{for } \tau \geq T_z \geq T_p \\ T_z/T_p & \text{for } T_p \geq T_z \geq 5\tau \\ \frac{\tilde{T}_p/T_p}{\tilde{T}_p - T_z} & \text{where } \tilde{T}_p = \min(T_p, 5\tau) \geq T_z \end{cases} \tag{6}$$

$$\frac{K_p}{(1+T_1 s)(1+T_2 s)(1+T_3 s)(1+T_4 s)} \approx \frac{K_p \cdot e^{-(T_3/2+T_4)s}}{(1+T_1 s)[1+(T_2+T_3/2)s]}$$

1.2 SOPDT model

Second Order Plus Dead Time

$$W(s) = \frac{Ke^{-\tau \cdot s}}{(1+T_1 \cdot s)(1+T_2 \cdot s)} \tag{7}$$

- gain, two time constants, delay
- used if $T_2 > \tau$
- setting of the parameters is not an easy task!

1.3 Empirical model: identification

Theoretical models, based on chemistry and physics of the process, may not be practical for complex processes if model requires large number of equations with a lot of process parameters and unknown parameters. An alternative approach is to develop an empirical model directly from experimental data. In general, empirical dynamic models are simpler than theoretical ones [5].

If we would like to tune the control loop, it is more likely you will design a model by performing a plant test.
Process model is based on experimental data:

Test, observation of the I/O signals - "black box"

✓ Planning

Model objectives: that is, how will be used, who will be the user?

A priori information: stable, static;

Operating point, the input (step response, value), what to measure;

How much time it takes, safety requirements.

✓ Test

Presence of the non-linearity.

Are other inputs stable?

✓ Structure of the model: what is known?

Aim is to obtain model with acceptable accuracy.

What is needed to control the process.

CRI - Control Relevant Identification.

✓ Parameters estimation

Graphical, statistical (regression analysis).

✓ Evaluation of the model

How accurately the model describes the data?

✓ Verification

Additional check with other data.

Models Types

model structure	test type				model parameters
	step	impulse	2 impulses	PRBS	
1. order	X	X			K_p, T
2. order	X				$K_p, T/\omega, \beta$
FOPDT	X				K_p, T, τ
SOPDT	-				K_p, T_1, T_2, τ
ARX	-			X	

Step response or process reaction curve

- Often used, easy to understand;
- Does not contain information about the high-frequency behavior;
- Does not have theoretical advantages;
- Works well with a noise; if signal/noise ratio <5 it is hard to found the derivative.
- Not perfect step change in the input causes the error

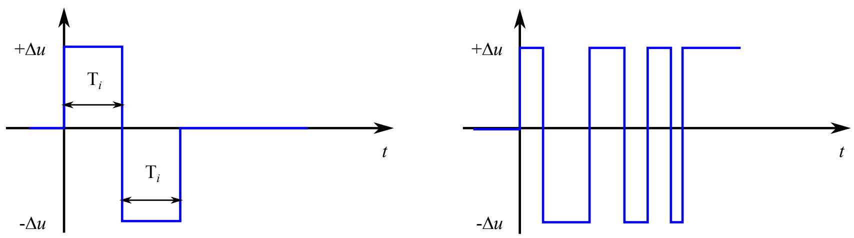
  rise time $t_s = 0.1 \cdot \tau$ of the input signal causes the measurement error of a delay τ up to 20%.

The input must be changed enough to observe a change in the output variable (it must increase above the noise level), but not so much that the output change is too great (economical reasoning) [1].

Simple identification tests provide simple process models, which can be used to design the control system with limited features. Sometimes that is enough.

Other methods

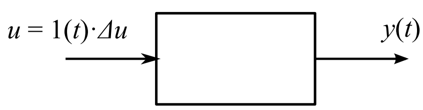
- Two impulses
- Pseudo-Random Binary Sequence (PRBS)



(a) Two impulses (b) PRBS

Figure 1: Test signals

1.4 Experimental estimation of the FOPDT model parameters



step response of a static object

input step change Δu at time instance 0

Register a reaction curve in response to a step change in the input from one steady-state value to another $y_1 \to y_2$ (see Fig. 2)

The process gain K_p

$$K_p = \frac{\Delta y}{\Delta u} = \frac{y(t \to \infty)}{u(t \to \infty)} \tag{8}$$

Maximum Slope Method

Locate the inflection point of the process reaction curve and draw a tangent (puutuja / касательная) along it. The intersection of the tangent line and the time axis y_1 corresponds to the estimate of time delay $\tau = [0\ A]$. The intersection with the final steady-state line helps to calculate the time constant $T = [A\ B]$.

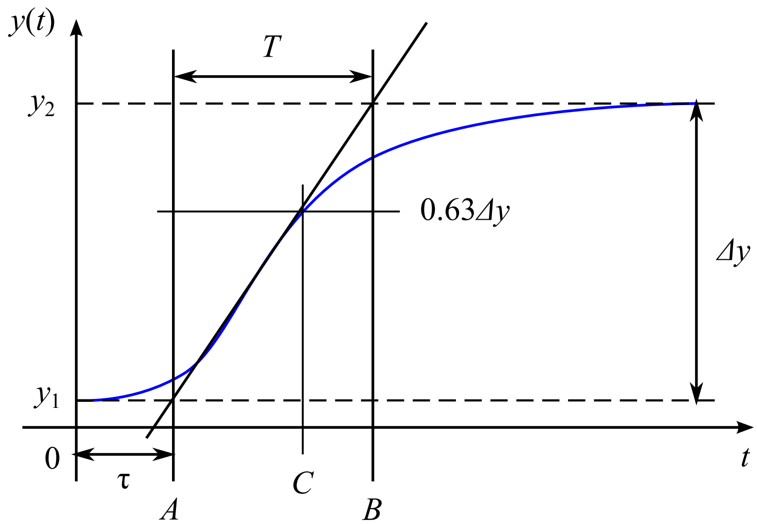


Figure 2: Process reaction curve

Numerical Application of 63% Method

Time delay can be found like in previous method: $\tau = [0\ A]$. Time constant T can be obtained by looking at the 63% response time. Calculate the $0.63 \cdot \Delta y$ of the output signal. Mark the time instance then output value is equal to it, so $T = [A\ C]$. More accurate technique.

Two-Point Method

Observes the output relative change $y'(0 \ldots 100\%)$
$y' = (y - y_1)/(y2 - y_1)$. Here the time required for the process output to make 28.3% and 63.2% of the long-term change is denoted by $t_{28.3\%}$ and $t_{63.2\%}$, respectively. The time constant and time delay can be estimated the following way

$$T = 1.5(t_{63.2\%} - t_{28.3\%}) \tag{9}$$

$$\tau = t_{63.2\%} - T \tag{10}$$

In case of the 20% and 80% levels

$$T = 0.721(t_{80\%} - t_{20\%}) \tag{11}$$

$$\tau = 1.161 t_{20\%} - 0.161 t_{80\%} \tag{12}$$

The primary limitation to using step responses to identify FOPDT transfer functions is the amount of time required to assure that the process is approaching a new steady state. That is,

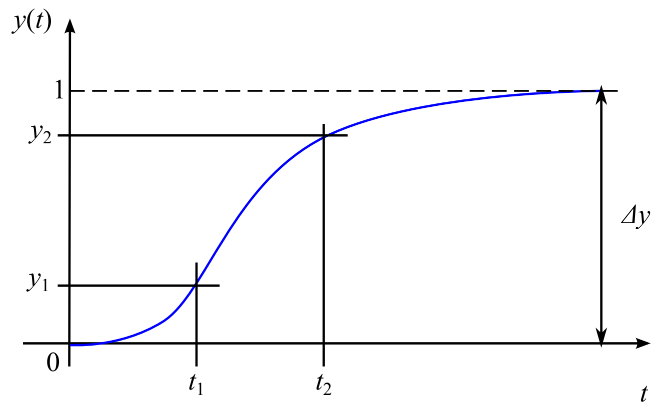


Figure 3: Two-Point Method

the major limit is the time required to determine the gain of the process. For large time constant processes it is often desirable to use a simpler model that does not require a long step test time [1].

Bibliography

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