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TALLINN UNIVERSITY OF TECHNOLOGY

Fractional-order Process Model based Control Design

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Talk overview

The main goal of my PhD thesis is to design and implement a fractional-order PID-type controller capable of efficient and reliable self-tuning for robust industrial process control. In this talk we shall consider the following topics:

- Mathematical basis of fractional-order calculus;
- Transition from conventional process models to those of fractional-order;
- The FO-FOPDT model: analysis of frequency-domain characteristics;
- $PI^{\lambda}D^{\mu}$ controllers, robustness and design specifications;
- Application of Newton's method to $PI^{\lambda}D^{\mu}$ controller tuning.

Introduction: Mathematical Basis of Fractional-order Calculus

Introduction: Historical facts

- The concept of the differentiation operator $\mathscr{D} = \mathrm{d}/\mathrm{d}x$ is a well-known fundamental tool of modern calculus. For a suitable function f the n-th derivative is well defined as

$$\mathscr{D}^n f(x) = \mathrm{d}^n f(x)/\mathrm{d}x^n, \tag{1}$$

  where n is a positive integer.

- What happens if we extend this concept to a situation, when the order of differentiation is arbitrary, for example, fractional?

- That was the very same question L'Hôpital addressed to Leibniz in a letter in 1695. Since then the concept of fractional calculus has drawn the attention of many famous mathematicians, including Euler, Laplace, Fourier, Liouville, Riemann, Abel.

Fractional derivative of a power function: An approach based on intuition

For the power function $f(x) = x^k$ the fractional derivative can be shown to be

$$\frac{\mathrm{d}^\alpha f(x)}{\mathrm{d}x^\alpha} = \frac{\Gamma(k+1)}{\Gamma(k-\alpha+1)} x^{k-\alpha}, \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function—generalization of the factorial function—defined as

$$\Gamma(x) = \int_0^\infty t^{x-1} \mathrm{e}^{-t} \mathrm{d}t, \quad x > 0. \tag{3}$$

For example,

$$\frac{\mathrm{d}^{1/2}(x^2)}{\mathrm{d}x^{1/2}} = \frac{\Gamma(3)}{\Gamma(5/2)} x^{3/2} = \frac{8x^{3/2}}{3\sqrt{\pi}}.$$

Basics of fractional calculus

Fractional calculus is a generalization of integration and differentiation to non-integer order operator ${}_a\mathscr{D}_t^\alpha$, where a and t are the limits of the operation and $\alpha \in \mathbb{R}$ is the fractional order

$$
{}_a\mathscr{D}_t^\alpha = \begin{cases} \frac{\mathrm{d}^\alpha}{\mathrm{d}t^\alpha} & \Re(\alpha) > 0, \\ 1 & \Re(\alpha) = 0, \\ \int_a^t (\mathrm{d}\tau)^{-\alpha} & \Re(\alpha) < 0. \end{cases} \tag{4}
$$

The following relation holds for noninteger exponentiation of the imaginary unit j and is frequently encountered in fractional calculus. We shall make extensive use of it as illustrated in the later part of the talk.

$$
j^\alpha = \cos\left(\frac{\alpha\pi}{2}\right) + j\sin\left(\frac{\alpha\pi}{2}\right) \tag{5}
$$

Fractional-order derivative definitions: Laplace transform

Definition 1. *(Riemann-Liouville)*

$$\mathscr{L}\left[{}^{R}\mathscr{D}^{\alpha}f(t)\right] = s^{\alpha}F(s) - \sum_{k=0}^{m-1} s^{k}\left[\mathscr{D}^{\alpha-k-1}f(t)\right]_{t=0}. \qquad (6)$$

Definition 2. *(Caputo)*

$$\mathscr{L}\left[{}^{C}\mathscr{D}^{\alpha}f(t)\right] = s^{\alpha}F(s) - \sum_{k=0}^{m-1} s^{\alpha-k-1}f^{(k)}(0). \qquad (7)$$

Definition 3. *(Grünwald-Letnikov)*

$$\mathscr{L}\left[{}^{GL}\mathscr{D}^{\alpha}f(t)\right] = s^{\alpha}F(s). \qquad (8)$$

For the first two definitions we have $(m-1 \leqslant \alpha < m)$.

Approximation of fractional operators: The Oustaloup filter

The Oustaloup recursive filter gives a very good approximation of fractional operators in a specified frequency range and is widely used in fractional calculus. For a frequency range (ω_b, ω_h) and of order N the filter for an operator $s^\gamma, 0 < \gamma < 1$, is given by

$$
\begin{aligned}
s^\gamma &\approx K \prod_{k=-N}^{N} \frac{s + \omega'_k}{s + \omega_k}, \quad K = \omega_h^\gamma, \quad \omega_r = \frac{\omega_h}{\omega_b}, \\
\omega'_k &= \omega_b (\omega_r)^{\frac{k+N+\frac{1}{2}(1-\gamma)}{2N+1}}, \quad \omega_k = \omega_b (\omega_r)^{\frac{k+N+\frac{1}{2}(1+\gamma)}{2N+1}}.
\end{aligned} \tag{9}
$$

The resulting model order is $2N + 1$.

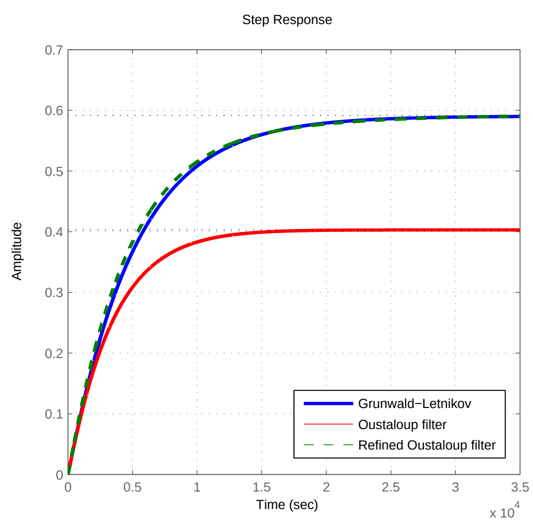
A modified Oustaloup filter has been proposed in literature [3].

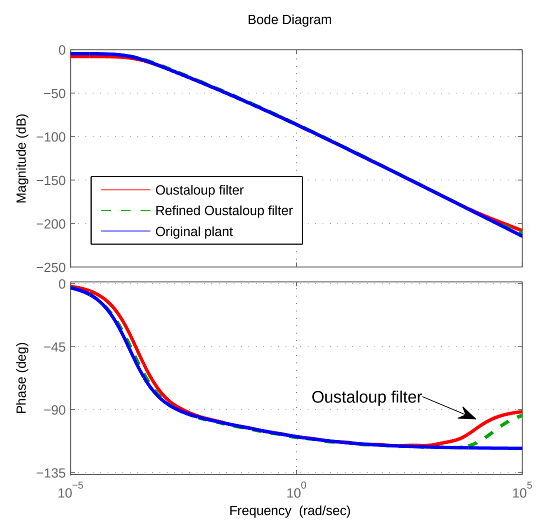
Oustaloup filter approximation example

Recall the fractional-order transfer function

$$G(s) = \frac{1}{14994s^{1.31} + 6009.5s^{0.97} + 1.69},$$

and approximation parameters $\omega = [10^{-4}; 10^{4}]$, $N = 5$.





Transition to Fractional-order Process Models

Fractional-order Process Models

Author's contribution:

- Investigation of fractional-order characteristics of industrial plant models;
- Development of suitable identification methods for FO process models.

Selected publications:

1. A. Tepljakov, E. Petlenkov, and J. Belikov, "FOMCON: a MATLAB toolbox for fractional-order system identification and control," *International Journal of Microelectronics and Computer Science*, vol. 2, no. 2, pp. 51–62, 2011
2. A. Tepljakov, E. Petlenkov, J. Belikov, and M. Halás, "Design and implementation of fractional-order PID controllers for a fluid tank system," in *Proc. 2013 American Control Conference (ACC)*, Washington DC, USA, June 2013, pp. 1780–1785

Under review:

3. A. Tepljakov, E. Petlenkov, and J. Belikov, "Closed-loop identification of fractional-order models using FOMCON toolbox for MATLAB," in *Proc. 14th Biennial Baltic Electronics Conference*, 2014, submitted for review

Fractional-order transfer functions

A linear, fractional-order continuous-time dynamic system can be expressed by a fractional differential equation of the following form

$$\begin{aligned} a_n \mathscr{D}^{\alpha_n} y(t) + a_{n-1} \mathscr{D}^{\alpha_{n-1}} y(t) + \cdots + a_0 \mathscr{D}^{\alpha_0} y(t) &= \\ b_m \mathscr{D}^{\beta_m} u(t) + b_{m-1} \mathscr{D}^{\beta_{m-1}} u(t) + \cdots + b_0 \mathscr{D}^{\beta_0} u(t), \end{aligned} \tag{10}$$

We apply the Laplace transform with zero initial conditions and obtain the fractional-order transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \cdots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \cdots + a_0 s^{\alpha_0}}. \tag{11}$$

In the case of a system with commensurate order q we have

$$G(s) = \frac{\sum\limits_{k=0}^{m} b_k \left(s^q\right)^k}{\sum\limits_{k=0}^{n} a_k \left(s^q\right)^k}. \tag{12}$$

Process models

Consider the following generalizations of conventional process models used in industrial control design.

(FO)FOPDT	$G(s) = \frac{K}{1+Ts}\mathrm{e}^{-Ls}$	$G(s) = \frac{K}{1+Ts^\alpha}\mathrm{e}^{-Ls}$
(FO)IPDT	$G(s) = \frac{K}{s}\mathrm{e}^{-Ls}$	$G(s) = \frac{K}{s^\alpha}\mathrm{e}^{-Ls}$
(FO)FOIPDT	$G(s) = \frac{K}{s(1+Ts)}\mathrm{e}^{-Ls}$	$G(s) = \frac{K}{s(1+Ts^\alpha)}\mathrm{e}^{-Ls}$

The system is modeled in continuous time in the following way:

$$\dot{x}_1 = \frac{1}{A}u_1 - d_{12} - w_1 c_1 \sqrt{x_1}, \qquad (13)$$

$$\dot{x}_2 = \frac{1}{A}u_2 + d_{12} - w_2 c_2 \sqrt{x_2},$$

where x_1 and x_2 are levels of fluid, A is the cross section of both tanks; c_1, c_2, and c_{12} are flow coefficients, u_1 and u_2 are pump powers; valves are denoted by $w_i : w_i \in \{0, 1\}$ and

$$d_{12} = w_{12} \cdot c_{12} \cdot \mathrm{sign}(x_1 - x_2)\sqrt{|x_1 - x_2|}.$$

Real-life example: Fractional-order control of the coupled tank system (continued)

Our task is to control the level in the first tank. We identify the real plant from a step experiment with $w_1 = w_{12} = 1, w_2 = 0$ in (13). The resulting fractional-order model is described by

$$G(s) = \frac{2.442}{18.0674 s^{0.9455} + 1} \mathrm{e}^{-0.1s}. \qquad (14)$$

We notice, that this model does not tend to exhibit integer-order dynamics. Due to the relatively small value of the transport delay the basic tuning formulae for integer-order PID tuning do not provide feasible results.

Analysis of the Fractional-order First-Order Plus Dead Time (FO-FOPDT) Model

Fractional-order FOPDT process model

Author's contribution:

- Investigation of applicability of the FO-FOPDT model to industrial control problems;
- Application of the Newton-Raphson method to the problem of deriving plant characteristics in the frequency domain;
- Study of FO-FOPDT model characteristics and development of suitable tuning rules based on the model with the goal of implementing a FOPID controller with autotuning capabilities.

This is a work in progress. Currently a paper is under review:

1. A. Tepljakov, E. Petlenkov, and J. Belikov, "Optimization based robust FOPI and FOPID controller design for fractional FOPDT plants in embedded control applications," in *2014 IEEE 53rd Annual Conference on Decision and Control (CDC)*, 2014, submitted for review

And several more manuscripts are in preparation.

The modified Newton-Raphson method

Consider the problem of finding a root ω^* of a general nonlinear equation $f(\omega) = 0$ under the constraints $\omega > 0$ and $\omega \in (\omega_b, \omega_h)$. To tackle the problem one may employ the Newton-Raphson method which usually provides quadratic convergence to the solution. The process of locating the root starts at an initial guess ω_0 and is given by the following iterative formula:

$$\omega_{k+1} = \omega_k - f(\omega) \left(f'(\omega) \right)^{-1}. \tag{15}$$

Once a prescribed iteration limit ν is reached, or the necessary tolerance ϵ is achieved under the condition $f(\omega_k) < \epsilon$, the algorithm shall stop returning the root ω^*. However, there is a drawback of this algorithm such that local minima of $f(\omega)$ may lead to the change of sign of $f'(\omega)$ and a violation of the condition $\omega_m > 0$ at iteration step $m = k + 1$ may occur. To rectify this, the locally obtained solution at step n may be replaced such that $\omega_n = \gamma_c \omega_k$, where $\gamma_c \neq 1$ is some predefined positive factor. If as a result of this modification ω_m no longer belongs to the interval (ω_b, ω_h), the process shall fail returning $\omega^* = 0$ thereby indicating that it could not find a solution.

The modified Newton-Raphson method: Algorithm summary

```
procedure NEWTON(ω_0, γ_c, ω_b, ω_h, f, f')
    ε ← Tolerance, ν ← MaxIterations
    k ← 0; ω_k ← ω_0
    while k < ν and f(ω) > ε do
        ω_{k+1} ← ω_k − f(ω_k)(f'(ω_k))^{−1}
        if ω_{k+1} < 0 then
            ω_{k+1} ← γ_c · ω_k
        end if
        if ω_{k+1} < ω_b or ω_{k+1} > ω_h then
            return 0
        end if
        k ← k + 1
    end while
    return ω_k
end procedure
```

The FO-FOPDT process model

Recall, that the FO-FOPDT model is given by the following transfer function

$$G(s) = \frac{K\mathrm{e}^{-Ls}}{Ts^{\alpha} + 1}, \tag{16}$$

where it is assumed that $K > 0$, $T > 0$, $L > 0$ and $\alpha \in (0, 2]$. We suppose that all of the parameters of this plant are known *a priori*. They may be obtained, for instance, by employing an identification procedure of a real life process. We begin the analysis by deriving the equations to obtain the magnitude and phase angle of $G(j\omega)$. This is done by replacing $s = j\omega$ in (16), employing (5), and isolating the real and complex parts of the resulting expression as $z = a + jb$. Then, the magnitude A and phase angle φ are simply computed as $A = \sqrt{a^2 + b^2}, \varphi = \tan^{-1}(b/a)$.

The FO-FOPDT process model: Magnitude and phase response

Magnitude:

$$|G(j\omega)| = \frac{|K|}{\sqrt{1 + T^2\omega^{2\alpha} + 2T\omega^{\alpha}\cos\left(\frac{\alpha\pi}{2}\right)}} \tag{17}$$

Phase angle:

$$\arg\left(G(j\omega)\right) = -L\omega - \tan^{-1}\left(\frac{T\sin\left(\frac{\alpha\pi}{2}\right)}{\omega^{-\alpha} + T\cos\left(\frac{\alpha\pi}{2}\right)}\right). \tag{18}$$

The obtained relations will be used in the following calculations.

The FO-FOPDT process model: Gain crossover frequency and phase margin

Next we derive open-loop characteristics of this plant. We begin by obtaining the gain crossover frequency ω_c, for which it holds

$$|G(j\omega_c)| = 1.$$

Solving this equation yields

$$\omega_c = \left(\frac{\sqrt{K^2 + \cos^2\left(\frac{\alpha\pi}{2}\right) - 1} - \cos\left(\frac{\alpha\pi}{2}\right)}{T} \right)^{1/\alpha}. \tag{19}$$

The phase margin φ_m of the system can then be determined from

$$\varphi_m = \pi - \arg\left(G(j\omega_c)\right) + 2\pi n, \quad n \geqslant 0. \tag{20}$$

The FO-FOPDT process model: Phase crossover frequency and gain margin (1)

It is more difficult to derive a formula to find the phase crossover frequency, also referred to as the ultimate frequency of the system ω_u, since we need to solve a transcendental equation

$$-L\omega_u - \tan^{-1}\left(\frac{T\sin\left(\frac{\alpha\pi}{2}\right)}{\omega_u^{-\alpha} + T\cos\left(\frac{\alpha\pi}{2}\right)}\right) = -\pi - 2\pi n, \qquad (21)$$

where n is determined by the requirement to obtain a minimum gain margin $1/\left|G(j\omega_u)\right|$ closest to unity. While ω_u is usually obtained during relay autotuning, if it is not given, then the following method may be used to compute it from the FFOPDT model parameters. We first introduce a function

$$v(\omega) = \arg\left(G(j\omega)\right) + \pi + 2\pi n. \qquad (22)$$

The FO-FOPDT process model: Phase crossover frequency and gain margin (2)

We compute the derivative $\mathrm{d}\upsilon(\omega)/\mathrm{d}\omega$. After simplification we arrive at

$$\upsilon'(\omega) = -L - \frac{\alpha T \sin\left(\frac{\alpha\pi}{2}\right)}{\omega\left(2T\cos\left(\frac{\alpha\pi}{2}\right) + \omega^{-\alpha} + T^2\omega^{\alpha}\right)}. \quad (23)$$

We may now use the modified Newton's method to obtain ω_u. Note, that to locate the minimum stability margin we need to introduce a modification to the search algorithm, whereby instead of terminating upon obtaining a solution ω_u^* the gain margin $1/\left|G(j\omega)\right|$ at this frequency is checked. If it is found to be less than unity, the iterative process is repeated assigning $\omega_g \leftarrow \omega_u^*$, $\omega_0 \leftarrow \omega_u^*$ and $n \leftarrow n+1$. This means that the search direction must be positive. The gain margin are then determined by means of

$$K_c = \min\left(\left|1 - 1/G(j\omega_g)\right|, \left|1 - 1/G(j\omega_u)\right|\right). \quad (24)$$

Note, that the search interval $\omega \in (\omega_b, \omega_h)$ is related to the band-limited Oustaloup approximation of a suitable fractional-order controller.

Tuning of Fractional-order Controllers

Tuning of Fractional-order Controllers

Author's contribution:

- FO-FOPDT model based robust $PI^{\lambda}D^{\mu}$ controller design due to frequency-domain specifications;
- Research of auto-tuning methods for $PI^{\lambda}D^{\mu}$ controllers and implementation thereof in a hardware prototype.
- Application of Newton's method for systems of nonlinear equations to $PI^{\lambda}D^{\mu}$ controller tuning.

This is a work in progress. Results are to be published.

Fractional-order controllers

The fractional PI$^\lambda$D$^\mu$ controller, where λ and μ denote the orders of the integral and differential components, respectively, is given by

$$C(s) = K_p + K_i s^{-\lambda} + K_d \cdot s^{\mu}. \tag{25}$$

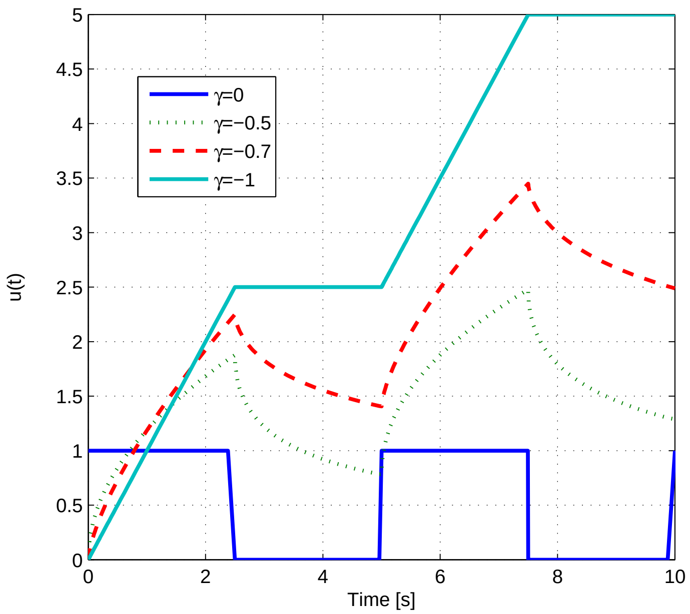
The transfer function, corresponding to the fractional lead-lag compensator of order α, has the following form:

$$C_L(s) = K \left(\frac{1+bs}{1+as} \right)^{\alpha}. \tag{26}$$

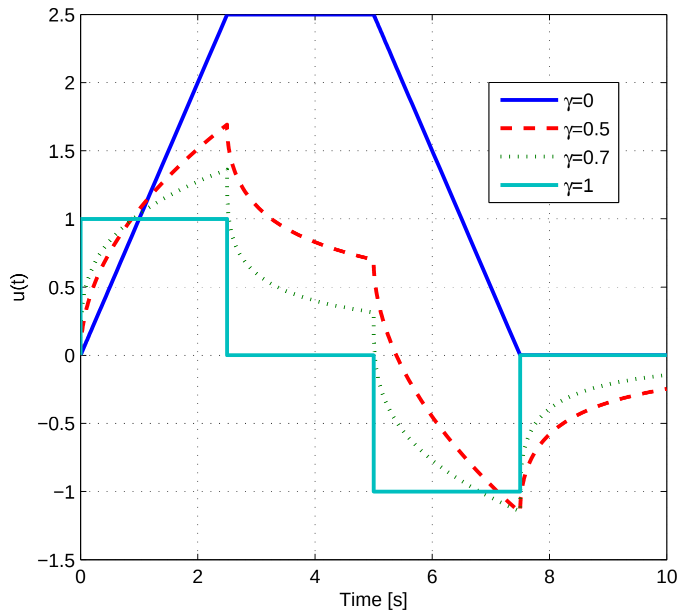
When $\alpha > 0$ we have the fractional zero and pole frequencies $\omega_z = 1/b$, $\omega_h = 1/a$ and the transfer function in (26) corresponds to a fractional lead compensator. For $\alpha < 0$, a fractional lag compensator is obtained.

Basics of fractional control: fractional control actions

Let a basic fractional control action be defined as $C(s) = K \cdot s^{\gamma}$. The control actions in the time domain for $\gamma \in [-1, 1]$ with $K = 1$ under different input signals are given below.

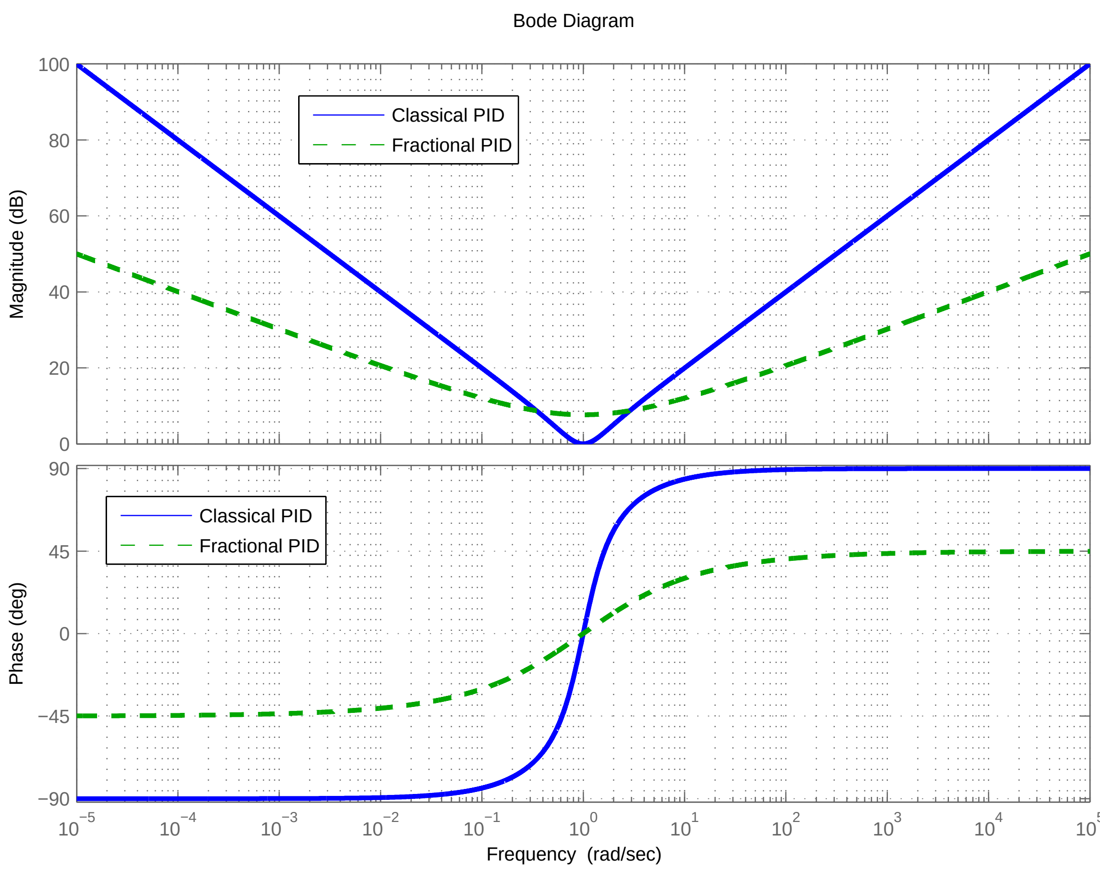


Fractional integrator $s^{-\gamma}$



Fractional differentiator s^{γ}

PID controller vs. $PI^{0.5}D^{0.5}$ controller: frequency-domain characteristics



Optimization based PI$^\lambda$D$^\mu$ tuning

Optimization provides general means of tuning a fractional-order PID controller given a cost function and suitable optimization constraints. There are several aspects to the problem of designing a proper controller using constrained optimization:

- The type of plant to be controlled (integer or noninteger order, nonlinear);
- Optimization criterion (cost function);
- Fractional controller design specifications;
- Specific parameters to optimize;
- Selection of initial controller parameters.

Optimization based PI$^\lambda$D$^\mu$ tuning: Constraints

The design specifications include:

- Gain margin G_m and phase margin φ_m specifications;
- Complementary sensitivity function $T(j\omega)$ constraint, providing A dB of noise attenuation for frequencies $\omega > \omega_t$ rad/s;
- Sensitivity function $S(j\omega)$ constraint for output disturbance rejection, providing a sensitivity function of B dB for frequencies $\omega < \omega_s$ rad/s;
- Robustness to plant gain variations:

$$\left(\frac{\mathrm{d}\arg C(j\omega)G(j\omega)}{\mathrm{d}\omega}\right)_{\omega=\omega_c} = 0, \tag{27}$$

  i.e. a flat phase of the system is desired within a region of the system critical frequency ω_c.

FOPID controller: Magnitude response

We derive the expression for the magnitude response as

$$|C(j\omega)| = \sqrt{C_R^2(\omega) + C_I^2(\omega)}, \tag{28}$$

where

$$C_R(\omega) = K_p + \omega^{-\lambda} K_i \cos\left(\frac{\lambda\pi}{2}\right) + \omega^{\mu} K_d \cos\left(\frac{\mu\pi}{2}\right) \tag{29}$$

and

$$C_I(\omega) = -\omega^{-\lambda} K_i \sin\left(\frac{\lambda\pi}{2}\right) + \omega^{\mu} K_d \sin\left(\frac{\mu\pi}{2}\right). \tag{30}$$

FOPID controller: Phase angle

The phase angle of the FOPID controller may be computed using

$$\arg\left(C(j\omega)\right) = \tan^{-1}\left(\frac{C_N(\omega)}{C_D(\omega)}\right), \tag{31}$$

where

$$C_N(\omega) = \omega^{\lambda+\mu} K_d \sin\left(\frac{\mu\pi}{2}\right) - K_i \sin\left(\frac{\lambda\pi}{2}\right) \tag{32}$$

and

$$C_D(\omega) = K_i \cos\left(\frac{\lambda\pi}{2}\right) + \omega^{\lambda}\left(\omega^{\mu} K_d \cos\left(\frac{\mu\pi}{2}\right) + K_p\right). \tag{33}$$

FO-FOPDT plant and FOPID controller: Open-loop characteristics

We can now derive the equations to compute the critical frequencies and corresponding stability margins of the open-loop control system given by $G_{ol}(j\omega) = C(j\omega)G(j\omega)$. A function $\psi_{pm}(\omega)$ for the phase margin is defined as

$$\psi_{pm}(\omega) := |C(j\omega)| \cdot |G(j\omega)| - 1 \tag{34}$$

To use the modified Newton-Raphson method to solve for ω_c the equation $\psi_{pm}(\omega_c) = 0$ we need to compute the derivative $\psi'_{pm}(\omega)$. In the same manner, define the function $\psi_{gm}(\omega)$ for the gain margin

$$\psi_{gm}(\omega) = \arg\left(C(j\omega)\right) + \arg\left(G(j\omega)\right) + \pi + 2\pi n \tag{35}$$

and take the derivative $\psi'_{gm}(\omega)$. Then solve $\psi_{gm}(\omega_u) = 0$ for ω_u. Finally, to check whether phase is flat at ω_c, ensuring the robustness to gain variations specification, one may check whether the following relation holds

$$\psi'_{gm}(\omega_c) = 0. \tag{36}$$

The equations to compute the sensitivity functions are derived in the same way.

Application of Newton's Method to FOPID Controller Tuning

Idea for tuning FOPID controllers for the FO-FOPDT model: Latest developments

The basic idea is this. We have five parameters to tune, out of which two are selected based on some concrete rule. The other three parameters must be optimized to satisfy three design specifications. Current implementation is as follows:

- Choose λ—the order of the fractional integrator—according to the F-MIGO rule;
- Choose μ—the order of the fractional differentiator—according to control system output signal measurement SNR (no concrete relation yet);
- Select three design specifications, form three equations for K_p, K_i and K_d—the FOPID controller gains—and use the Newton method to solve the system of these equations.

FOPID control for FO-FOPDT: F-MIGO

The F-MIGO method has been developed based on observations using a test batch of regular FOPDT models used for FOPI controller design. Based on the relative dead time parameter

$$\tau = \frac{L}{L + T} \tag{37}$$

the following rule was established in [3] for selecting the order of the fractional integrator

$$\lambda = \begin{cases} 1.1, & \tau \geqslant 0.6, \\ 1.0, & 0.4 \leqslant \tau < 0.6, \\ 0.9, & 0.1 \leqslant \tau < 0.4, \\ 0.7, & \tau < 0.1. \end{cases} \tag{38}$$

FOPID control for FO-FOPDT: The Newton method

We have $x = \begin{bmatrix} K_p & K_i & K_d \end{bmatrix}^\top$ and must solve for Δx the matrix equation

$$J \Delta x = -F. \tag{39}$$

The next iterate is computed as $x^+ = x + \Delta x$, where

$$J = \begin{bmatrix} \frac{\partial \kappa_1(\cdot)}{\partial K_p} & \frac{\partial \kappa_1(\cdot)}{\partial K_i} & \frac{\partial \kappa_1(\cdot)}{\partial K_d} \\ \frac{\partial \kappa_2(\cdot)}{\partial K_p} & \frac{\partial \kappa_2(\cdot)}{\partial K_i} & \frac{\partial \kappa_2(\cdot)}{\partial K_d} \\ \frac{\partial \kappa_3(\cdot)}{\partial K_p} & \frac{\partial \kappa_3(\cdot)}{\partial K_i} & \frac{\partial \kappa_3(\cdot)}{\partial K_d} \end{bmatrix} \tag{40}$$

is the Jacobian matrix and

$$F = \begin{bmatrix} \kappa_1(\cdot) & \kappa_2(\cdot) & \kappa_3(\cdot) \end{bmatrix}^\top \tag{41}$$

is the specifications vector. Both J and F are evaluated at the current value of x. Then $x \leftarrow x^+$ and the process continues until either the design goal is satisfied, or divergence or excess of allowed number of iterations is detected.

FOPID control for FO-FOPDT: Chosen specifications

The following specifications are considered:

- Gain crossover frequency ω_c;
- Phase margin φ_m (in radians) which is computed using knowledge of ω_c;
- Robustness to gain variations $\psi'_{gm}(\omega_c) = 0$.

The following functions are thus constructed:

$$
\begin{aligned}
\kappa_1(\cdot) &= |C(j\omega_c)| \cdot |G(j\omega_c| - 1, && (42)\\
\kappa_2(\cdot) &= \arg(C(j\omega_c)) + \arg(G(j\omega_c)) + \pi - \varphi_m, && (43)\\
\kappa_3(\cdot) &= \psi'_{gm}(\omega_c). && (44)
\end{aligned}
$$

FOPID control for FO-FOPDT: Example

Consider a plant

$$G(s) = \frac{66.16\mathrm{e}^{-1.93s}}{12.72s^{0.5} + 1} \tag{45}$$

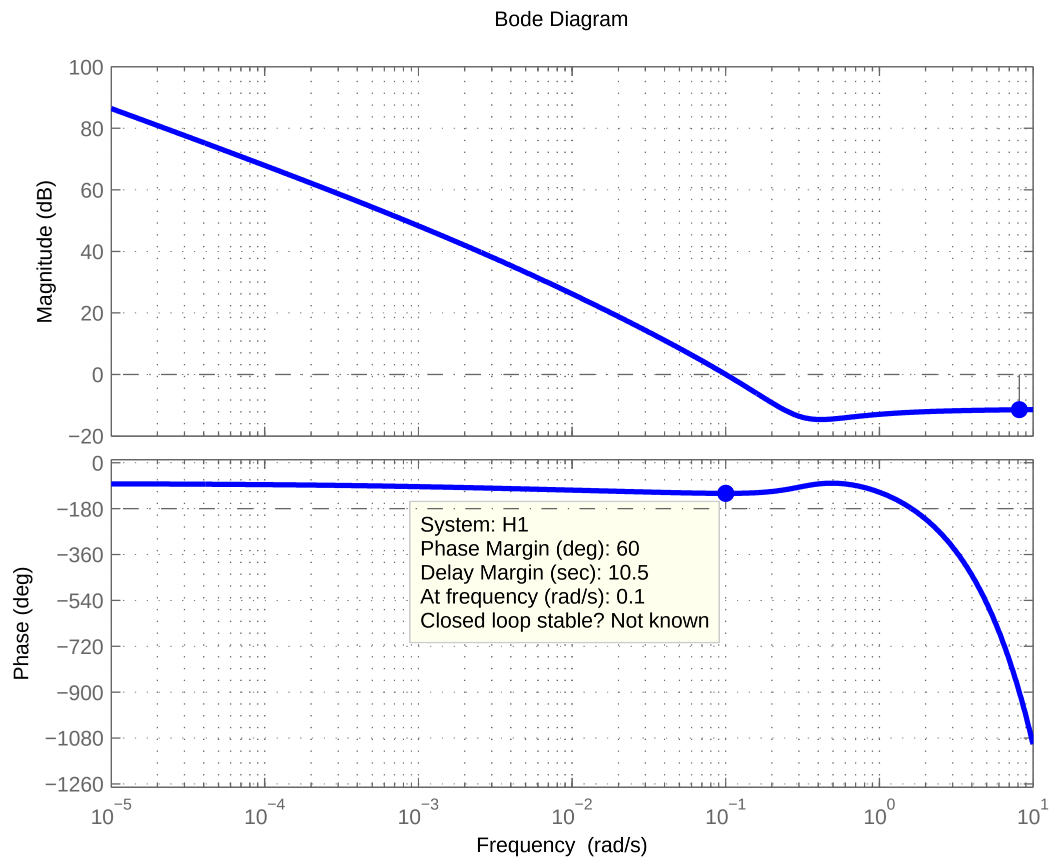
which represents a model of a heating process. In the following, we illustrate the controller design procedure. Specifications are $\omega_c = 0.1$, $\varphi_m = 60^\circ$, and $\psi'_{gm}(\omega_c) = 0$.

In this case, F-MIGO yields $\lambda = 0.9$ and we choose $\mu = 0.5$. For this problem we take the initial solution as

$$K_p = K_i = K_d = 1/K = 1/66.16 = 0.0151 \tag{46}$$

and apply Newton's method.

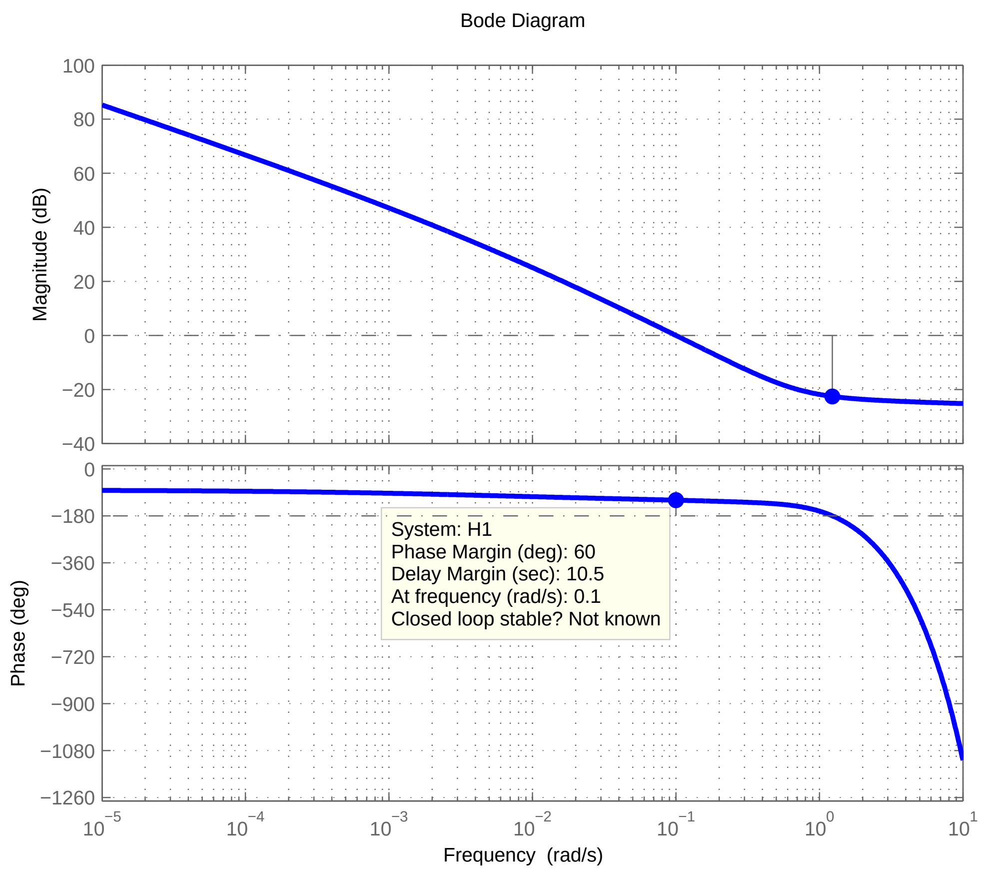
FOPID control for FO-FOPDT: Results (Newton's method, 5 iterations)



Discussion: Identified issues

- There exists the problem of selecting initial controller gains;
- Optimization is unconstrained in search variables—no control over signs of controller gains, although specifications are still satisfied;
- Computations are expensive;
- Computational floating-point stability when implementing on relatively low-end hardware is still to be investigated;
- Process diverges if the distance from solution is considerable.

FOPID control for FO-FOPDT: Preliminary results (Newton-Krylov method)



FOMCON project: Fractional-order Modeling and Control

FOMCON

- Official website: `http://fomcon.net/`
- Toolbox for MATLAB available;
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Discussion

Thank you for listening!